AN ANALYSIS OF LATERAL STRAIN IN CYLINDRICAL GRAIN BIN
WALLS SUBJECTED TO ACTIVE AND PASSIVE PRESSURES

By

HARVEY ELDON HAMILTON

Bachelor of Science in Agricultural Engineering

Oklahoma State University

Stillwater, Oklahoma

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Thesis Approved:

E. T. Wilson

Thesis Adviser

George W. A. Mahoney

J. H. Boggs

Dean of the Graduate School

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CHAPTER I

INTRODUCTION

During the past two decades, surplus grains have been stored in a variety of facilities including steel tanks which were previously designed and used for storage of liquids. Failures of some of these facilities, including steel storage structures manufactured by commercial firms, raises questions on design criteria and loads imposed on structures by stored grain.

Stresses in the walls of storage structures are a result of active and passive pressures. Active pressures are produced as a result of the weight of the grain. The design equations presently being used predict the active pressure which should be present at a particular depth below the surface of a prescribed grain mass. This predicted pressure in conjunction with the thin-walled cylinder theory is used to calculate the stress requirements for the wall. This procedure is sufficient for structures containing a liquid but is questionable for grain bins.

Passive pressures are produced in a grain storage structure when there is relative lateral movement of the sidewall and the grain mass. This relative movement can be the result of a change in the moisture content of the grain or a change in the temperature of the grain or the bin material. Vibration and settling of the grain also contribute to the passive pressures produced in a grain storage structure. Persons who have investigated the causes of some failures concluded that passive pressures did exist at the time of failure, however, the magnitude of these pressures were not known.

To determine the effects of active and passive pressures on strain or stress produced in a wall, it would be necessary to measure the strain directly and eliminate the necessity of applying the thin-walled cylinder theory after measuring the pressure. This is the purpose of this study.

The symbols used in this thesis are listed and defined in Appendix A, page 68.

CHAPTER II

REVIEW OF LITERATURE

Introduction

Since late in the eighteenth century when Coulomb derived an equation to express earth pressures, there has been considerable controversy about the actual pressures exerted by grains in storage bins. Isaac Roberts (1884) conducted an investigation on a large grain storage bin to determine horizontal and vertical pressures. The measuring device consisted of large pressure panels connected to a massive counterweight beam. Since this pioneer investigation of grain pressures, prototype and model bins of various conformations, dimensions, and characteristics have been employed to investigate the effects of pertinent variables on grain pressures. However, the factors of similitude between model and prototype bins remain to be confirmed.

Equations for Determining Grain Pressures

Theoretical equations have been formulated to predict pressures within and at boundaries of granular mass by Coulomb and Rankine for the general case, and by Janssen and Airy in separate approaches to the specific problem of grain storage. These equations are fully explained in Ketchum's text (1919).

Janssen's equation and Airy's equation are widely used for designing grain structures. The equations for the simple case are listed as follows; the symbols are defined in Appendix A, page 68.

(A) Janssen's

$$N = \frac{wR}{u'}\left[1 - e^{-\frac{ku'h}{R}}\right] \qquad (2.1)$$

(B) Airy's
(Shallow Bins)

$$N = wh\left[\frac{1}{\sqrt{u(u+u')} + \sqrt{1+u^2}}\right]^2 \qquad (2.2)$$

These equations give results which are on the safe side under normal conditions. However, Janssen's equation is not theoretically correct because it fails to consider the angle of repose of grain. Neither equation takes into consideration the surcharge present in most grain storages.

Rankine's equation is derived and fully explained in Hough's text (1957). Expressions for the active and passive pressure values for a level surface are as follows; the symbols are defined on page 68 and subscripts a and p refer to active and passive respectively:

$$N_a = wh\left[\frac{1 - \sin \phi}{1 + \sin \phi}\right] \qquad (2.3)$$

$$N_p = wh\left[\frac{1 + \sin \phi}{1 - \sin \phi}\right] \qquad (2.4)$$

When there is a sloping backfill, the equations take the form:

$$N_a = wh \operatorname{Cos} \theta \left[\frac{\operatorname{Cos} \theta - \sqrt{\operatorname{Cos}^2\theta - \operatorname{Cos}^2\phi}}{\operatorname{Cos} \theta + \sqrt{\operatorname{Cos}^2\theta - \operatorname{Cos}^2\phi}}\right] \qquad (2.5)$$

$$N_p = wh \operatorname{Cos} \theta \left[\frac{\operatorname{Cos} \theta + \sqrt{\operatorname{Cos}^2\theta - \operatorname{Cos}^2\phi}}{\operatorname{Cos} \theta - \sqrt{\operatorname{Cos}^2\theta - \operatorname{Cos}^2\phi}}\right] \qquad (2.6)$$

Experiments by Trepanier (1960) show that grain stored in a heaped or surcharge pattern produces lateral pressures which agree closely with the above equation for active lateral pressure.

Some Physical Properties of Grain

Values of k

The ratio of the lateral to the vertical pressures at any point is represented by the letter k. This is one of the values which must be determined before Janssen's equation can be applied. The value of k can be determined only from experiments, using the same materials which will be used in a particular storage structure. The range of k from a maximum to a minimum is usually determined and the maximum is used for design.

It was found in experiments reviewed by Ketchum (1919) that the value of k is not a constant. Plessiner found that k decreases with larger depths of grain; however, experiments by Williams and Ketchum agree that k increases with the depth of the grain.

Kramer (1944) plotted values of k for various total vertical loadings in pounds. This showed that k is not a constant, and it increases more rapidly at lower vertical loads.

Ketchum proposed that an estimate of k can be obtained by the following equation: $k = (1 - \sin \phi)/(1+\sin\phi)$ where ϕ = angle of repose of grain on grain.

Coefficient of Friction

The value of the coefficient of friction of grain on a bin surface can be affected by several factors. Balis (1959) found that surface accumulations, surface roughness, and the sample constraint produce varying effects depending upon many of the other parameters. Investigations by Balis, which dealt with relative effects, indicated that surface accumulations cause 15 per cent variations. Surface roughness variations on galvanized steel resulted in a 50 per cent variation in friction force. The results of this study show that the variation in the reported friction coefficients could be caused by variations in procedure alone.

Investigations by Lorenzen (1959) indicate that grain moisture content, pressure of the grain on the friction surface and the condition of the friction surface may product varying effects on the coefficient of friction. The coefficient of friction has several special meanings, depending on the method of determination and the use of the coefficient.

Two methods of determining the coefficient of friction were reviewed by Ketchum (1919). One method by Airy was to determine the angle at which a piece of the bin wall would slide down the grain slope. Jamieson used a cantilever type apparatus whereby a piece of the bin wall material was placed on one end of the cantilever with a shallow tray of grain inverted and placed on the material. The cantilever beam was balanced by a counterweight. By raising the pivoted frame, the angle of friction was taken when the tray began to move.

Kramer (1944) employed a similar method to the one used by Jamieson except a tilting-top drawing table was used instead of a cantilever beam. It was found that additional weight placed on top of the grain had no significant effect on the angle at which movement began.

Angle of Repose

The value of the angle of repose is dependent upon the method in which it is obtained. The angle of repose can easily be mistaken for the angle of flow. The angle of repose can be obtained by piling the grain or by allowing the grain to empty from the side of a container. There are variables which affect the piling angle, such as rate of flow and height of fall. Therefore, the emptying angle of repose is the better method.

The angle of flow can be obtained by placing grain in a shallow tray and tilting the tray to an angle at which the grain begins to move.

Investigations by Lorenzen (1959) showed that there is a significant difference between the values of angle of repose, angle of flow, and the internal angle of friction of respective grains at a particular moisture content. In the high moisture range (from 13 to 20 per cent), the influence of moisture on the mean coefficient is more pronounced and is especially apparent for wheat and corn. The coefficient of repose in this range increased by 31 per cent for wheat and by 20 per cent for corn. The angle of repose for milo is significantly higher than the general range for wheat, corn, and barley. This may be due to the stacking effect of spherical particles.

It was shown that a steel surface was unquestionably smoother than a wood surface; however, the coefficient for corn on wood was lower than for corn on steel.

Investigations by Kramer (1944) also showed that the angle of repose was greatly influenced by the moisture content, increasing very rapidly at higher moistures, especially when it exceeded 16 to 17 per cent.

Some Factors Affecting Pressure Distribution

Vibration

The effects of vibration on granular material were noted by Roberts (1884) while conducting experiments to determine grain pressures. He noted that the weight of a cubic food of wheat when filled loosely was less than the weight of the same volume of wheat that had been shaken and pressed into the measure.

While conducting experiments with models, Jamieson reported that by tapping the bin with a hammer near the bottom, the pressure on the bottom could be decreased. Also, when the tapping was continued from the bottom to the top on all sides, the grain settled 2 to 3 inches and the bottom pressure was slightly increased.

Moore (1952) conducted tests in a rectangular model bin with both loose and vibrated material. The pressures were compared with water pressures. Fluid-pressure factors were computed by the relation:

$$F = \frac{Sg}{Sw} \times \frac{w}{g} \tag{2.7}$$

Where Sg is the stress (or deflection) produced by the granular material; Sw is the stress produced by water; w is the weight of water; and g is the weight of the granular material. Thus F is the ratio of the pressure produced by the granular material to the pressure produced by the same depth of a liquid of equal weight.

There was total settlement of as much as two to three inches from the level starting depth of 30 inches. It was observed that gravel and cement are apparently more susceptible to such vibration than wheat, which was generally the more "fluid" in its behavior.

Vibration of the test bin resulted in definite increases in bulging pressures over those observed in the initial static loading. It is significant that the mean fluid-pressure factor indicated for wheat after vibration was about 50 per cent greater.

Stresses and deflections under a 28 inch depth of wheat exceeded in many cases those obtained from a 28 inch depth of gravel weighing almost twice as much.

The findings indicate that a lateral fluid-pressure factor of $F = 0.60$ should be assumed for all the granular materials considered, where it is evident that impact or vibration may be encountered.

The tests by Moore indicate that the deflection of the bottom was not nearly as marked as those observed for the sides of the bin. All tests indicate that the deflection due to water was greater in all cases than the deflections caused by the granular materials.

Reynolds (1954) reported that the settlement of soils due to vibrational loading is many times greater than that produced by an equivalent static load, and its magnitude is dependent upon the frequency. The greatest settlements in soil are produced within a range of 300 to 2000 cycles per minute and within this range lie the natural frequencies of compressors and gas and diesel engines.

Vibrational or slow repetitional loads, dependent upon their point of application and direction, can produce six types of displacement. One is in a vertical direction, two are in horizontal directions at right angles to each other, and there are three rotational displacements about the three axes.

Time

During Lufft's experiments, reviewed by Ketchum (1919), the filling was interrupted at different intervals for various lengths of time. It was noted that the lateral pressure decreased for several hours or days before it became constant. It was surmized that the increase in pressure was due to the settlement of the grains, so that both the angle or repose and the coefficient of friction of the grain on the bin walls are increased.

Kramer (1944) reported that during some experiments, the test bin was not emptied until twenty-four hours after filling. In one instance a week elapsed before it was emptied. In each case, when a delay occurred before emptying began, it was noted that the maximum bin pressure occurred immediately after filling and then decreased for a period of time before becoming constant.

Saul (1953) reported that during a series of experiments three different bin loads were observed for periods of several days, two for 18 days and one for 25 days. The indicated pressure on the floor panels increased with each reading for 18 days in all three cases. The indicated pressures were the same between the 18th and the 25th day. The assumption that the weight of the grain in the section of the bin directly above the floor panels should be measured by the sum of the vertical pressure readings of the floor panels plus the vertical load carried by the walls at both ends of the floor panel section seemed valid. A close approximation to this vertical wall load would be twice the sum of the vertical wall pressure readings on the wall panels at one end of the floor panel section. In two cases of uniform filling of the bin, this assumption was not borne out until the 18th day after filling.

Ketchum (1919) reported that maximum lateral pressures occur immediately after filling, and are slightly greater in a bin filled rapidly than in a bin filled slowly.

<u>Moisture Content Affects</u>

It has been generally recognized for several years that moisture content can affect pressure distribution in stored grain. However, very little research had been conducted to determine this affect until 1954 when Dale and Robinson (1954) conducted experiments to determine the effect of moisture content on pressures in deep storage bins. Their experiments were conducted in a five foot high cylindrical test bin made of 14-gage sheet steel rolled to a diameter of 18 inches. Air conditioning equipment was used to provide high humidity air to pass through the dry (14.0 per cent wet base) grain. It was possible to obtain relative humidities of 83 per cent, thus raising the moisture content of the corn to 17 per cent wet basis.

This experiment led to the following conclusions:

1. Increases in the moisture content of stored corn from 1 to

4 per cent during ventilation produced stresses in the sidewalls of a deep bin at least six times that of dry grain.

2. An increase in the moisture content of ten per cent by flooding will develop pressures as high as ten times that of dry grain.

3. As the pressure increases due to swelling, the pressure distribution on the sidewalls approaches a straight line similar to that of a liquid.

4. Where grains increase in moisture content, Janssen's equation is not sufficiently accurate for computing lateral pressures.

Lorenzen (1960) conducted experiments to determine the moisture effect on ratio of principle pressures in stored grain. Triaxial tests were conducted on grain samples to determine the k factor. It was found that the unit weight of granular material decreases as the moisture content is increased. With this in mind, an examination of the equations derived by Rankine, Coulomb, Airy, and Janssen indicates that as the moisture content increases, the lateral pressure decreases.

The tests indicate that an increase in moisture content increases the angle of repose and decreases the value of k, the ratio of lateral to vertical pressure. Lorenzen concluded that vertical wall loads are critical when grain moisture content is lower than the usual equilibrium moisture content in most localities.

<u>Temperature Change and Wall Movement</u>

Huntington and Lutzelschwab (1936) found that the movement of a pressure cell away from the grain of as little as 0.005 inches resulted in a marked decrease in the pressure reading, and that when the cell was pushed back to its original position the pressure reading was much greater than the original value.

Weiland (1962) investigated the causes of structural failures of grain storage facilities. It was concluded that a temperature drop rate of 8 degrees or more per day, coupled with a low of less than +15 degrees may be the cause of some structural failures. Based on the assumption that the grain does not compact as the steel shell shrinks, the calculated stresses are as follows:

Shrink Stresses in Steel Shell = ECT

where E = Modulus of elasticity of the steel.

C = Coefficient of expansion or contraction of the steel per degree temperature change in degrees Fehrenheit.

T = Temperature change in degrees Fahrenheit.

The product of C x T will give expansion or contraction of steel in inches per inch of length. The change in diameter will be the same, that is, inches per inch of diameter.

If the grain does compact as the diameter decreases, the stress in the steel would be a function of the ratio of the modulus of compression of the grain to the modulus of elasticity of the steel.

Many ductile steels pass into a brittle region at temperatures encountered during the winter months. Should a small break or weak spot develop in a structure and the sum of assorted stresses reaches a dangerous value, then it may require only a sudden shock, such as a gust of wind, a hammer blow, a passing train or vehicle or a similar force to cause a rupture in the steel and collapse the structure.

Weiland concluded that lap welds should not be used in grain bin structures at either girth or longitudinal seams. This applies particularly in the lower rings of the tank and in metals 1/4 inch thick or greater. Metal plates 1/4 inch or thinner very rarely behave in a brittle manner.

Failures in storage bins have originated at all levels between the first ring at the floor plate and rings which are at levels more than half the height of the tank.

Henderson (1941) conducted an experiment to determine the thermal expansion of shelled corn in mass. A steel cylinder, 6 inches in diameter and 12 inches high was fitted with a welded bottom, a plate to rest upon the top surface of the corn, and a dial gauge to observe the up and down motion of the plate due to expansion of the corn. Observations were made with 10 inches of corn at a moisture content of 9.3 per cent in the cylinder and at temperatures of 77 degrees and 28 degrees. A linear coefficient of 0.0608×10^{-4} (°F) was used to correct the observed results for the expansion of steel in the container used. The linear coefficient of thermal expansion of shelled corn in mass was found to be 0.187×10^{-4} (F).

Henderson observed that during the winter the corn cooled, shrank, and settled and higher temperatures caused the corn to expand faster than the bin.

Pressure Distribution

Side Wall Pressures

Investigations by Saul (1953) indicate that the lateral wall pressures for all depths of fill in each of 15 loadings showed a decrease in pressures just above the floor. Observation of the friction pressure of corn on the floor panels showed a large increase near the walls. It appears that the lateral wall pressure is relieved by the lateral pressure supported on the floor in the same manner as the vertical floor pressure is relieved by the vertical load supported on the wall.

It has been shown by experiments that the lateral pressures increase as the depth increases, however, the increase in lateral pressure is much less after the height of grain exceeds approximately two and one-half times the diameter.

Factors affecting pressure distribution are discussed under a separate topic in this report.

Floor Pressures

It was observed in investigations by Isaac Roberts (1884) that the pressure increase upon the bottom ceases in bins at a point not exceeding in height two diameters of its inscribed circle. Several investigations in later years substantiate this observation and it is generally accepted that the increase of pressure ceases when the height of grain exceeds two to two and one-half times the diameters.

Saul (1953) conducted investigations to determine the pressure distribution in rectangular and square bins. It was shown in all tests that a decrease in floor load occurs in the area next to the walls. This is due to vertical load supported by the walls.

Ross and Isaac (1961) conducted experiments in a model bin to check the validity of their equation for predicting floor pressures. Experimental values agreed closely with calculated values. However, these values were considerably lower than experimental values obtained by Saul.

Effects of Loading

Investigations conducted by Saul (1953) indicate that the method of loading affects pressure distribution. The floor pressures are higher under the points where the grain is loaded into the bin even though the grain is leveled.

In one test, the grain was spouted into the bin just above the wall opposite the wall with the pressure panels and allowed to flow toward the panels. This caused a tilting of the walls and pulled the wall-pressure panels into the grain, resulting in very high readings due to the passive resistance of the grain.

The pressure distribution was relatively symmetrical when the bin was filled in two foot depth increments with a moving spout to keep the surface level without moving or sliding the grain after it was placed.

Instrumentation

Types Previously Used

Roberts (1884) employed pressure panels placed in the sides and the bottoms of prototype and model bins to determine grain pressures. The panels were of various sizes and were connected to graduated levers. Pressures were difficult to measure due to the apparent movement required to raise the lever.

Ketchum (1919) reviewed experiments conducted up to 1919. Several pressure measuring devices were devised but some were lacking in precision and pertinent properties of the grain were not always considered. Tolts placed a steel plate across an opening in a wall. The plate was held rigidly at two ends and the top and bottom were free to move. The deflection of the plate for various heights of grain was measured. Jamieson used hydraulic pressure diaphragms connected to a mercury gage. On model bins the diaphragm covered the entire bottom. It was also used to determine side pressures. Lufft used hydraulic pressure diaphragms filled with glycerine and connected to a mercury column measuring device. Kramer also used this type device in 1944. Pleissner determined the bottom pressures in bins by (1) weighing the movable bottom directly, (2) determining the deflection of the bottom and (3) placing rubber bags filled with water on the bottom of the bin. Williams measured side pressures by means of determining the amount of electric current flow through carbon plates at different pressures.

Ketchum conducted experiments in a model bin in 1919. A pressure plate was placed in the side of the bin and was mechanically linked to a platform scale by means of levers. Bottom pressures were determined directly by a platform scale.

Huntington and Lutzelschwab (1936) designed a pressure cell whereby the torque required to overcome the friction and thus turn a rotor, which was placed between a pressure diaphragm and the stationary back of the cell, was indicative of the pressure exerted on the pressure diaphragm. The cell was very economical to construct, easy to use and obtained favorable results. Eccentric loading did not affect the calibration values. However, temperature did affect the device considerably.

Fordham (1937) constructed a pre-loaded spring pressure measuring device to measure active and passive pressures. A spring balance was used to develop a selected pressure in the four springs located immediately behind the pressure plate. A dial gauge was used to determine the movement of the pressure plate against and away from the grain.

McCalmont (1938) used 3 1/2 feet wide by 4 feet high pressure panels made of 1 x 6 inch cribbing nailed to 2 x 4 inch cross pieces to measure pressures in corn cribs. The panels were held in place by two heat-treated steel bars, one near the top and one near the bottom. Pressure on the panel caused deflection of the bars which was measured by an instrument which would indicate the deflection between two points on the bar. The same type pressure panel was used by Saul (1953) to determine pressure distribution in rectangular grain bins.

Amundson (1945) ran experiments on a 2250-bushel capacity bin which was formed by 15 panels, each 4 feet by 10 feet, held in place by 7 steel bands. Thirty strain gages were placed at symmetrical locations on the metal bands and readings taken at different heights of grain. The strain in the band in microinches per inch multiplied by the modulus of elasticity of the steel determined the unit stress in the band.

Caughey and Tooles (1951) conducted experiments in a reinforced concrete cylinder 5 feet high and with 18 inches inside diameter. Steel plates, bent to the curvature of the inside surface of the bin, were fitted loosely into wall openings. The plates were supported by a half-inch rod projecting outward and clamped to cantilever bars, which in turn were welded to a steel ring at the base of the bin. The steel ring was firmly fixed to prevent movement. Thin stainless steel bands, approximately one-half inch wide, were placed around the outside of the bin and the cantilever bar at each opening. These bands were placed under tension by the use of turnbuckles. Strain gages were then placed on the steel bands and were calibrated by pulling on the plate with a horizontal force of known amount. Thin sheet rubber, in a slack condition, was placed over each opening to prevent dust and grain particles from wedging between the plates and the wall. The coefficient of friction between the pressure plates and the sheet rubber was determined by laying the rubber over a mound of sand and obtaining the force required to start the plate moving with a spring balance. The coefficients of friction between the sheet rubber and the materials being tested were also determined.

Moore (1952) conducted experiments in a rectangular model bin constructed primarily of aluminum sheeting. Lateral deflections of the wall were measured by means of a dial indicator and a suitable reference bar and by means of mirrored scales attached to the member in question with a fine reference wire stretched between its end. Vertical deflections of the bottom were measured with a dial indicator. Strains were determined by use of strain gages and corresponding stresses were computed by use of the modulus of elasticity of the material. A value of Poisson's ratio equal to one-third was used where strains at a point were measured in two directions, 90 degrees apart, and where it was necessary to consider biaxial stress relationships.

Dale and Robinson (1954) conducted experiments in a cylindrical bin five feet high made of 14-gage sheet steel rolled to a diameter of 18 inches. Pressure cells specially designed and machined to fit the curvature of the bin were placed in a spiral around the bin at intervals of approximately 12 inches vertically and 72 degrees horizontally. Pressure cells were also placed on the floor of the bin.

Trepanier (1960) conducted experiments in the field on large metal flat bulk grain storage bins. On one structure type A-3 BLH wire gages were Duco-cemented at 77 different locations on rigid frames, columns, ties, girts, struts, and side-wall panels. Wall deflections were measured from the outside at 83 different positions to supplement strain data. On another structure, the main interest centered on determination of tie rod loads. Special gage-mounted links were used to measure loads in the tie rods. The gages were protected from grain pressures by loosely fitted steel shields extending from clevis to clevis.

Collins (1962) conducted experiments in a cylindrical bin 12 feet high and 3.8 feet in diameter constructed of three sheets of "Building Aluminum" 4 x 12 feet and 0.032 inches thick. Strain gages were placed at various heights on the bin. Local bending was present in the thin walls so gages were placed on both the inside and outside at each position to separate the direct strain from the bending strain. Vertical and horizontal (or hoop) strain was measured inside and outside at each position requiring four gages at each of the fourteen positions. All gages were coated with Ten-X waterproofing compound and inside gages further coated with an epoxy resin to protect them from damage by the loading material. Stress was calculated from strain using Hook's Law for a biaxial state of stress:

$$S_a = \frac{E}{1 - u^2} \quad (e_a + ue_b) \tag{2.8}$$

where E = Modulus of elasticity

u = Poisson's ratio = 0.33

$e_{a,b}$ = Strain in directions a and b which are at right angles in microinches per inch.

Stress values for inside and outside membranes were averaged. In general, this method of strain measurement is precise to $\pm$ 10 microinch per inch giving an approximate precision of stress of $\pm$ 150 psi.

<u>Arching of Grain Over Pressure Measuring Device</u>

In the experiments reported by Ketchum (1919) there was some controversy on the effect of grain arching over pressure cells. Pleissner stated that the grain arches over small pressure surfaces thus a small pressure measuring surface does not give absolutely correct results, however, they do show relative changes correctly. This conclusion was drawn from the results of using three rubber diaphragms to determine lateral pressures. Experiments by Bovey, Jamieson, and Ketchum (1919) show that the arching action is insignificant, and that the pressures obtained with small and large pressure surfaces are in remarkable close agreement. Also, the results obtained with model bins agree very closely with the results obtained with full-sized bins.

Other investigators have indicated that arching of grain may have affected results obtained in their experiments, however, the smallest effective pressure measuring surface has not been determined.

CHAPTER III

THE STUDY

Objectives

The study was undertaken with the following objectives:

1. Determine the strains in the walls of cylindrical wheat storage structures due to active pressures as affected by:

    a. Depth below the grain surface

    b. Height of the bin

    c. Diameter of the bin

    d. Bulk grain density

    e. Stiffness index of the bin wall.

2. Determine the strains in the walls of cylindrical wheat storage structures due to passive pressures as affected by:

    a. Depth below the grain surface

    b. Height of the bin

    c. Change in the diameter of the bin.

3. Determine the elastic behavior of wheat when subjected to a compressive stress similar to that produced when there is relative lateral movement of the wall and the grain.

Assumptions and Limitations

It was assumed that the results obtained from the model study would be applicable to other geometrically similar grain storage structures. The validity of this assumption depends upon the correct application of the principles of similitude and the selection of the pertinent quantities.

Due to the time factor, the study was limited to hard red winter wheat which was maintained at a constant moisture content.

Pertinent Quantities

The physical quantities believed to be pertinent to the strain produced in the walls of cylindrical grain storage structures are listed in Table I. A discussion of some of these pertinent quantities follows.

Previous experiments have borne out the fact that the diameter (D) and the height of a bin affect the pressure distribution on the side walls of the bin. More specifically, the diameter-height ratio is pertinent to the storage system. It was found that the lateral pressure ceases to increase after the depth of grain exceeds twice the diameter.

For the same reasons stated in the prior paragraph, the depth below the surface of grain (h) is pertinent to the grain storage system. Both the height of the bin and the depth below the surface of the grain are pertinent to the strain produced in the wall.

The change in the diameter (d) of the bin represents the reaction of a prototype structure when the temperature is changed, or the expansion or contraction of grain when the temperature or the moisture content changes.

The characteristic particle length (l) describes the size of the particles (wheat grains) being stored in a structure. It is believed that the particle size affects the strains produced in bin walls.

The stiffness index (Et) determines the amount and character of elastic deformations of the wall. The wall and grain mass deformations thus interact to establish some equilibrium grain pressure and wall strain.

The elastic compression index of grain en masse (E_c) is a measure of

TABLE I

PERTINENT QUANTITIES

No	Symbol	Description	Units	Dimensional Symbol
1.	ϵ	Unit Strain In The Wall	Ft/Ft	——
2.	D	Diameter Of Bin	Ft	L
3.	H	Height Of Bin	Ft	L
4.	d	Change In Diameter Of Bin	Ft	L
5.	h	Depth Below Grain Surface	Ft	L
6.	ℓ	Characteristic Particle Length	Ft	L
7.	Et	Elastic Stiffness Index Of Bin Wall	Lb_F/Ft	FL^{-1}
8.	ρ	Bulk Mass Density Of Grain	Lb_M/Ft^3	ML^{-3}
9	E_c	Elastic Compression Index Of Grain In Mass	——	——
10.	G	Gravitational Field Strength	Lb_F/Lb_M	FM^{-1}
11.	m	Moisture Content Level	Percent	——
12.	ϕ	Angle Of Repose	Degrees	——
13	u'	Coefficient Of Friction Of Grain On Bin Wall	——	——
14.	S	Characteristic Shape Of Grain Particle	——	——
15.	α	Coefficient Of Volumetric Expansion Due To Temperature Change	$Degrees^{-1}$	ψ^{-1}
16.	η	Coefficient Of Volumetric Expansion Due To Moisture Change	——	——
17.	Δ	Temperature Change	Degrees	ψ

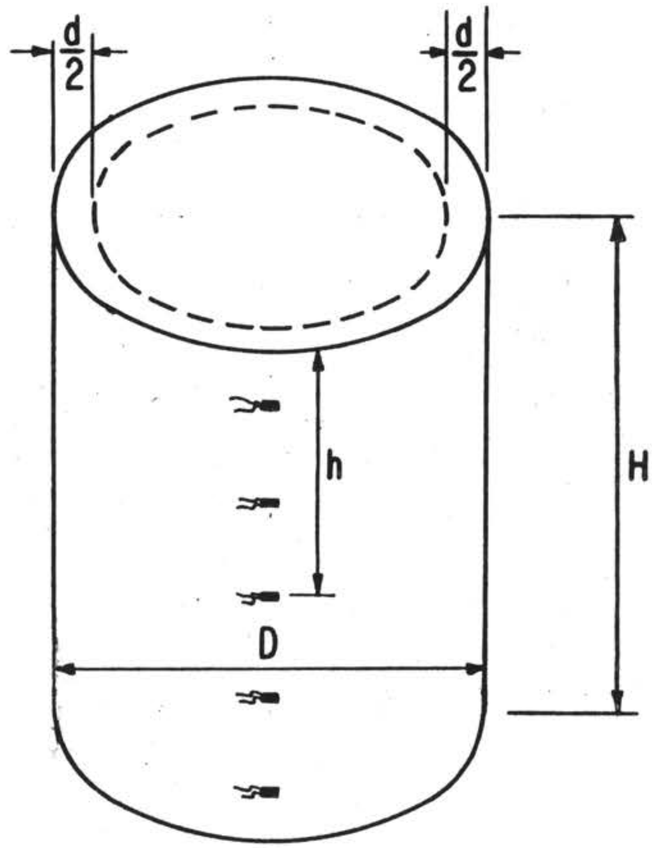


Figure 1. Definition Sketch of System.

the strain produced by an uniaxially applied stress on a grain mass which is subjected to a retaining pressure. This retaining pressure is produced by the weight of the grain. The elastic compression index influences the pressure produced against the wall of a grain bin and in turn affects the strain in the wall.

Grains of different types will have particles which are shaped differently. The shape of the particles will affect the pressure produced on a bin wall and must be included in the pertinent quantities.

The coefficients of volumetric expansion are indices of the amount of expansion of a grain mass due to temperature and moisture content changes while the grain is subjected to a retaining pressure. This retaining pressure is the result of the weight of the grain at a particular depth in the bin. The amount of expansion of the grain influences the pressure produced against a wall and thus affects the strains in the wall.

The gravitational field strength (G) will determine the weight of a mass of grain and must be included as a pertinent quantity for the strain produced in a grain bin wall.

Formation of Pi Terms

It is necessary to group the pertinent quantities into two groups to describe active pressures and passive pressures. This requires two equations for strain; one for active pressures and one for passive pressures.

Passive Pressures

There are seventeen pertinent quantities with four dimensions which describe this system. The rank of the dimensional matrix is four. Therefore, thirteen dimensionless Pi terms must be formed from the pertinent quantities. The Pi terms are as follows:

$\pi_1 = \varepsilon$	$\pi_5 = E_c$	$\pi_9 = \phi$
$\pi_2 = \frac{h}{H}$	$\pi_6 = \frac{d}{D}$	$\pi_{10} = u'$
$\pi_3 = \frac{D}{H}$	$\pi_7 = \frac{D}{\ell}$	$\pi_{11} = S$
$\pi_4 = \frac{\rho G H^2}{Et}$	$\pi_8 = m$	$\pi_{12} = \alpha\Delta$
		$\pi_{13} = \eta$

Active Pressures

The change in the diameter and the coefficients of volumetric expansion due to temperature and moisture content change are not pertinent to strains produced by active pressures. Therefore, for the system describing active pressures, π_6, π_{12}, and π_{13} can be omitted from the above list of Pi terms.

Experimental Design

The functional relationships among the Pi terms for passive and active pressures respectively are

$$\pi_1 = g(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10}, \pi_{11}, \pi_{12}, \pi_{13}) \quad (3.1)$$

$$\pi_1 = f(\pi_2, \pi_3, \pi_4, \pi_5, \pi_7, \pi_8, \pi_9, \pi_{10}, \pi_{11}) \quad (3.2)$$

Due to insufficient time to evaluate the influence of all the dimensionless parameters which influence the strain in a grain bin wall, experiments were conducted with wheat at a constant moisture content. The temperature of the grain was constant during the time the grain was in the model bin and the measurements of strain were being made.

The coefficients of friction of wheat on the galvanized steel and the aluminum were found to be essentially the same, therefore, π_{10} was considered to be constant for all tests.

It was hypothesized that, as $\pi_7 = D/\ell$ increases, the strains in a bin wall would approach a constant limiting value. It was further hypothesized that the values of π_7 for the models, and the prototypes which they represent, were within the range corresponding to the constant limiting value.

Equations 3.1 and 3.2 become

$$\pi_1 = g_1 (\pi_2, \pi_3, \pi_4, \pi_5, \pi_6) \tag{3.3}$$

$$\pi_1 = f_1 (\pi_2, \pi_3, \pi_4, \pi_5) \tag{3.4}$$

where the functions f_1 and g_1 can be determined from an analysis of experimental data.

The method of writing the prediction equation is to vary one independent Pi term while holding the others constant. The dependent Pi term, π_1, is measured and a component equation is established between the one varying Pi term and the dependent Pi term. This procedure is repeated for all other independent Pi terms. The resulting relationships between the dependent Pi term and the independent Pi terms can be combined to produce a general prediction equation for strain in a grain bin wall. A brief discussion of the Pi terms and the values selected for each follows:

$\pi_1 = \mathcal{E}$ is the horizontal strain produced in the wall. This is the dependent Pi term and was measured while the respective independent Pi terms are varied.

$\pi_2 = h/H$ is a parameter which specifies the position on the wall in which the strain is being measured. Five positions were selected for strain gage locations. The values for π_2 were 0.08333, 0.29167, 0.50, 0.70833, and 0.91667.

$\pi_3 = D/H$ was assigned values of 0.66667, 0.83333, 1.16667, and 1.50. These values includes bins sometimes regarded as shallow bins and some storage bins considered as deep bins.

$\pi_4 = \frac{\rho G H^2}{Et}$ is a parameter designed to determine the effect of the stiffness of the wall on strains produced by active and passive pressures. The range of variation of the parameter was limited due to the material available for bin construction. The tests were performed with bins having a diameter of 2.5 feet. The parameter was varied by using bins with different stiffness indices. The values of the parameter were 80.0×10^{-6}, 111.498×10^{-6}, and 167.3504×10^{-6}.

$\pi_5 = E_c$ is a function of the elastic properties of grain en masse and the retaining pressure to which the grain is subjected. An experiment designed to determine the value of this elastic index of grain was conducted with a tri-axial testing device. Retaining pressures were varied from five pounds per square inch to thirty pounds per square inch at five pounds per square inch increments.

A comparison of the retaining pressures used in the tests and the actual pressure produced at a particular depth in a prototype system could be made by using Janssen's or Rankine's equation to estimate the expected lateral pressure at a particular depth. This aspect should be investigated more fully by experimentation whereby pressure cells would be used to determine the actual pressure produced in a mass of grain at a particular depth in a prototype system.

The prediction equations derived from data obtained from this experiment can be used only for systems where the magnitude of each Pi term is within the range used for this experiment.

Table II shows a schedule of the experiments conducted in this study. Experiments 4 and 4a were conducted in one operation. The magniture of the parameter $\pi_5 = E_c$ is different for the two experiments.

TABLE II

SCHEDULE OF EXPERIMENTS

Parameter / Exp. No.	$\pi_1 = \epsilon$	$\pi_2 = h/H$	$\pi_3 = D/H$	$\pi_4 = \frac{\rho G H^2}{Et} \times 10^4$	$\pi_5 = E_c$	$\pi_6 = d/D$
1.	Measure	0.08333 0.29167 0.50000 0.70833 0.91667	Constant For Each π_3	Constant For Each π_4	0.7948	———
2.	Measure	Constant For Each π_2	0.66667 0.83333 1.16667 1.50000	1.673504	0.7948	———
3	Measure	Constant For Each π_2	0.83333	0.80 1.11498 1.673504	0.7948	———
4.	Measure	Constant For Each π_2	1.5	1.673504	0.7948	0 To .007
4a.	Measure	Constant For Each π_2	1.5	1.673504	0.2531	.007 To .0236

CHAPTER IV

EXPERIMENTAL EQUIPMENT

Model Bins

The model bins were constructed of metal sheeting three feet in width. After cutting the sheet to the correct length, the sheet was placed on edge and the ends were pulled together to form a model bin of the desired diameter. The ends were bolted together with two offset columns of one-eighth inch bolts.

A total of six model bins were required to perform the necessary experiments. Three bins were 2.5 feet in diameter. Thirty gage galvanized steel sheeting was used for one of these bins. Type 3003 aluminum with thicknesses of 0.02427 inches and 0.01617 inches were used for the two other bins.

The remaining three bins were constructed of aluminum sheeting with a thickness of 0.01617 inches. The diameters of these bins were 2.0 feet, 3.5 feet, and 4.5 feet.

All six bins were used to determine the effects of active pressures on the strain in the wall. Experiments to determine the effects of passive pressures were performed with the 4.5 foot diameter bin.

Device to Change the Diameter

The device shown in Figure 2 was used to change the diameter of the 4.5 foot bin. The sheet metal was bolted to both sides of the device. One end of the sheet metal was allowed to span the four inch gap between

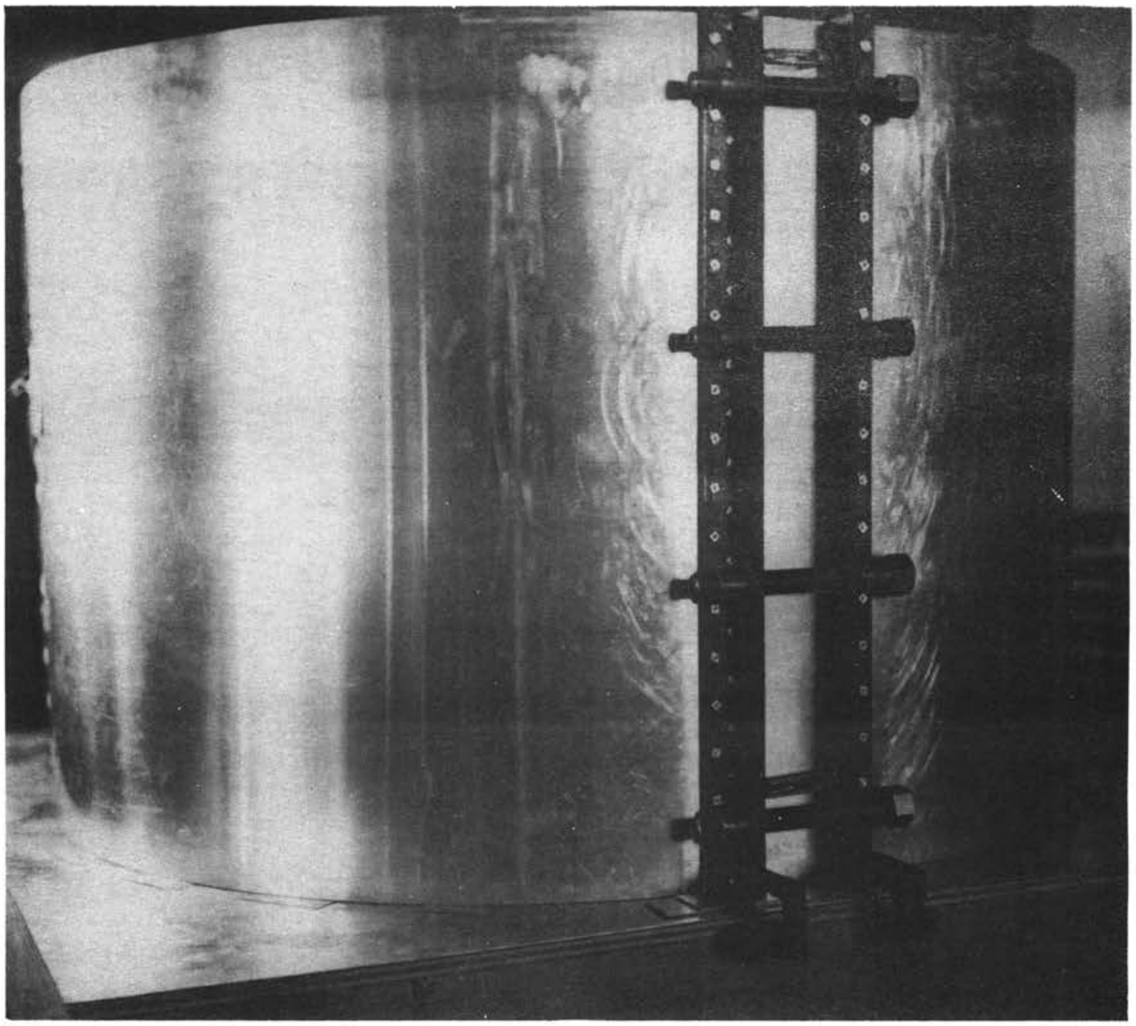


Figure 2. Device to Change the Diameter of the Bin.

the two sides of the device and overlap about three inches beyond on the inside of the bin. This free end was rolled to fit the curvature of the bin and was allowed to slide freely when the circumference was changed.

Preliminary experiments indicated the necessity of the caster wheels at the bottom of the device to prevent excessive lateral movement in that area when the diameter of the bin was decreased. Excessive lateral movement had caused the bin to move upward and had caused local distortion of the bin. Since there was no curvature in the device, the bin wall at the center of the device was placed to the inside of the scribed diameter line on the table top. This allowed the bin wall to be in the same relative position with respect to the initial diameter as the remainder of the bin after the device was fully closed.

The turnbuckles were used to set the initial opening to four inches and were removed after the bin was filled with grain. Four steel bolts one inch in diameter were used to close the gap in the device. These bolts were purposely over-designed to minimize twisting of the device when the circumference was changed. This provided more accuracy in measuring the changes in circumference. The incremental changes in circumference were measured with a micrometer.

The device rested on oiled shims which provided vertical alignment prior to bin loading and facilitated changing the circumference of the bin. Also, oiled shims were placed between the bin wall and the table top at approximately six inch intervals. This allowed movement of the wall with a minimum of bending near the bottom.

Work Table

The model bins were placed on a work table eighteen inches high and five feet square for the tests. The frame for the table top was constructed of two-by-six boards spaced 12 inches on centers. Five-eighths inch exterior plywood covered with aluminum sheeting 0.05 inch thick was used for the table top. The heavy construction was required to minimize deflection of the floor when the bins were loaded with grain.

Circles, the same diameters as the bins being tested, were scribed on the aluminum table top. The scribed lines were used to position the bins when preparing a test.

A three inch square hole was placed in the center of the table top and a sliding door was placed under the hole. This provided a means for emptying the bin after each test.

Grain and Grain Handling Equipment

Concho wheat was used in all experiments. This is a hard red winter wheat. The bulk density of the wheat was 49.86 pounds per cubic foot. The moisture content of the grain was 12.1 percent. The emptying angle of repose was 27.3 degrees. The coefficient of friction was 0.35 on galvanized steel and 0.33 on aluminum.

The grain was placed into a forced air grain drying bin to remove the excess heat and lower the moisture content immediately after harvesting. There was a small amount of trash and cracked grain present. The grain was believed to be a representative sample of wheat which is stored in cylindrical grain bin structures.

Approximately fifty bushels of this grain was placed in a small storage bin near the test equipment. A plastic sheet was placed over the storage bin when the grain was not being conveyed in an effort to maintain a constant moisture content in the grain.

The grain was elevated by a canvas belt conveyor to a small hopper located directly above the test bin. This conveyor minimized further cracking of the grain. The grain flowed from the hopper into the bin through a two inch square opening in the hopper at a rate of 3.02 cubic feet per minute. Discharge from the hopper was at bin height and directly in the center of the bin. The hopper could be elevated to allow leveling of the grain after the bin was filled.

Strain Gage Equipment

SR-4 strain gages were used to measure the horizontal strain in the wall at various depths. The gages used were Baldwin-Lima-Hamilton FA 50-12. An SR-4 strain indicator and a 20-channel switching-balancing unit were used to measure strain.

Since it was impossible to construct a bin with a constant curvature at all points, it was necessary to measure local bending in the bin wall. This was accomplished by placing strain gages on the outside and on the inside of the bin wall, directly opposite to each other. The direct strain was the average of the sum of the readings on opposing gages.

A column of strain gages consisted of a vertical array of five gages on both the inside and the outside of the bin. Two columns of gages were placed on each bin. The columns were placed ninety and one hundred eighty degrees from the position the bin was bolted together. A total of twenty gages were used for each model bin.

It is recognized that there would also be vertical strains produced by bending and direct stresses. Since these strains do not contribute greatly to the horizontal strains in small grain bins and since design criteria are based primarily on the horizontal strains produced in a bin, the vertical strains were not measured.

Temperature compensating gages were placed on two strips of bin material rolled approximately to the curvature of the bin. The strips of

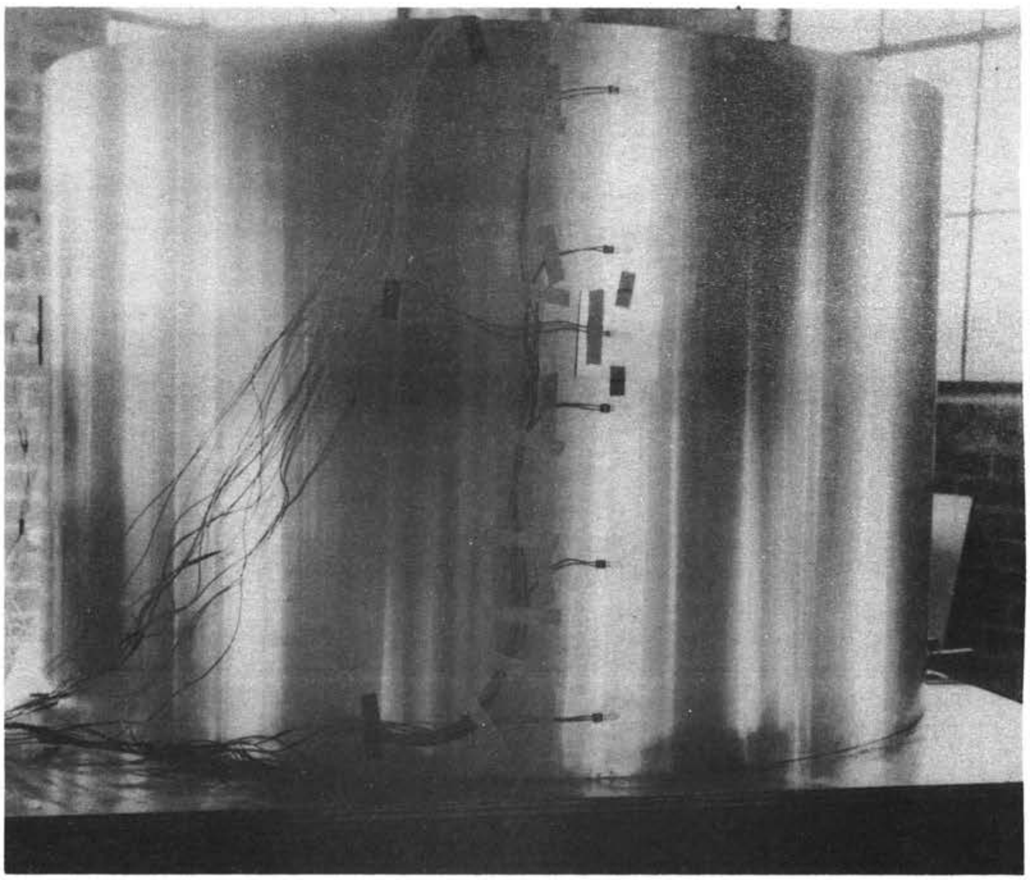


Figure 3. Strain Gage Arrangement.

metal were then loosely attached by two strips of tape to the outside of the bin near the corresponding column of strain gages. See Figure 3. These gages were utilized for the gages on both the inside and the outside of the bin. The gages on the inside of the bin were in closer contact with the grain. To minimize possible errors due to this, an effort was made to maintain the room air temperature at approximately the same temperature as that of the grain.

It was necessary to keep the grain from pressing directly against the strain gages on the inside of the bin. To accomplish this, small V-shaped arches were placed over the gages. The arches were attached to the bin wall with plastic electrical tape. The tape was quite elastic and offered little or no resistance to the strain in the wall.

Triaxial Testing Equipment

The equipment shown in Figure 4 was used to determine the elastic compression index of grain en masse. The Fairbanks-Morse platform scale was used to apply the load to the specimen. The mechanical advantage of the scale was one-hundred to one.

The tri-axial test cylinder was connected to the water reservoir-pressure cylinder by a heavy duty pressure hose and a valve. The pressure tank was used to obtain the desired air pressure in the water reservoir-pressure cylinder. When the valve was open between the reservoir and the test cylinder, the pressures equalized in both cylinders.

The vacuum tank was utilized in preparing the specimens for testing. The vacuum tank was connected to the inside of the test specimen by means of a hole in the base of the test cylinder.

The microdial measured the vertical displacement of the specimen and the clock was used to determine the time intervals for loading and for reading the deformation.

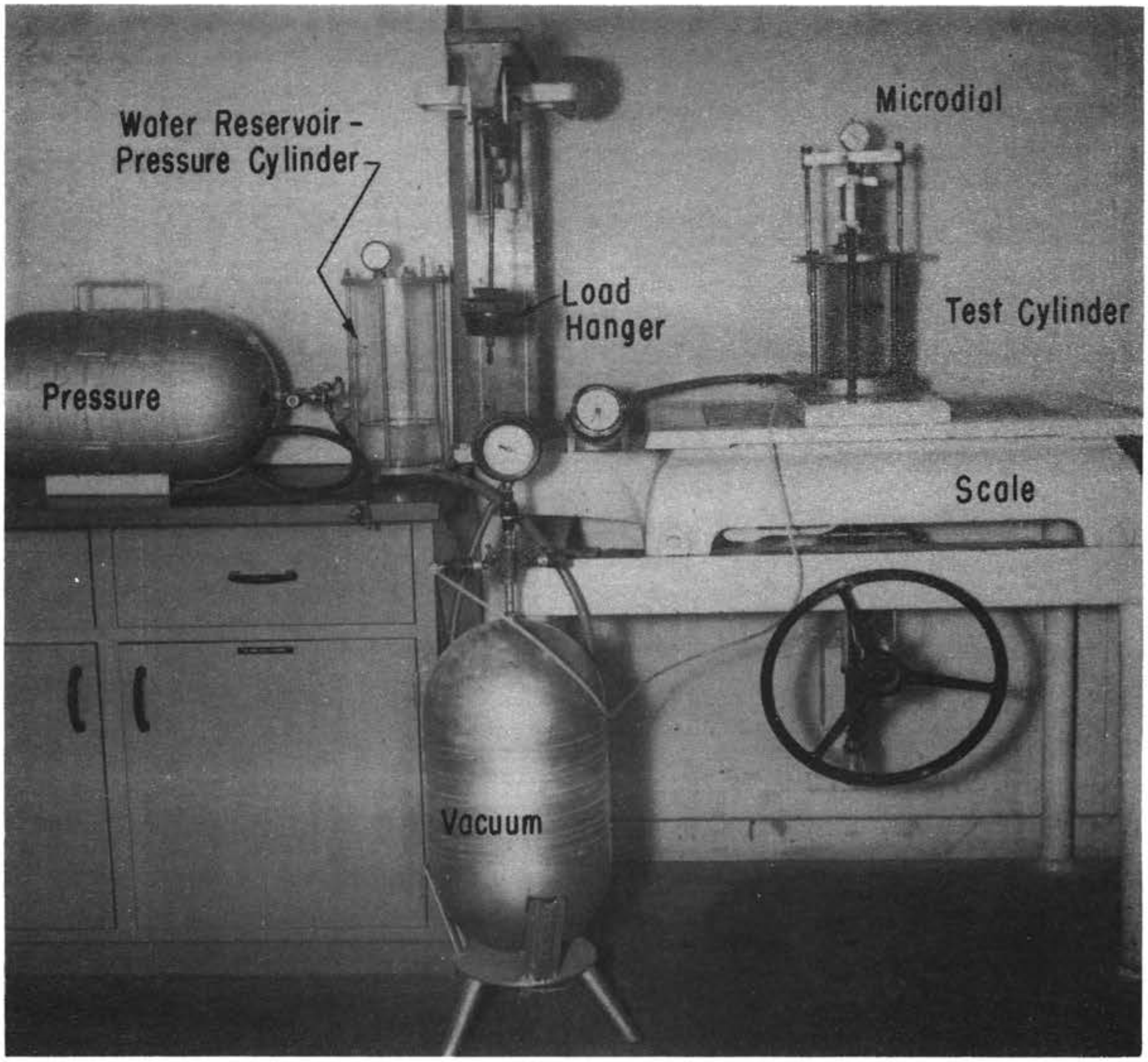


Figure 4. Triaxial Testing Equipment.

CHAPTER V

PROCEDURE

Determination of Strain Due to Active and Passive Pressures

The procedures for measuring strains produced by active pressures and passive pressures were similar in most respects. The model bin was positioned on the work table by aid of the lines scribed on the aluminum surface. The switching-balancing unit was set to an initial reading of 1000 microinches per inch for each strain gage. The grain was then elevated to the hopper and allowed to flow freely from the hopper into the center of the model bin. The grain was allowed to pile until the flow from the hopper was stopped. The grain was then leveled and again allowed to pile until the flow from the hopper was stopped. The grain was leveled again and this procedure was followed until the bin was completely full. Flow from the hopper was essentially continuous throughout the filling of the bin. A straight edge was used to level the grain to bin height.

The strains due to active pressures were measured at each of the twenty gages immediately after filling. The grain was then removed from the bin except for a small amount around the bin wall. This grain was not removed in order that the bin would not be disturbed from the position in which the initial strain gage readings were taken. This allowed the initial reading to be checked after the bin was unloaded.

The above procedure was followed in performing four replicated tests for each bin.

To obtain the strains due to passive pressures, the 4.5 foot diameter bin was positioned on the work table and the device to change the circumference was positioned to an opening of four inches by aid of the modified turnbuckles. The strain indicator was set to an initial reading of 1000 microinches per inch on each strain gage.

The grain was loaded into the bin in the same manner as for tests with active pressures. Strain readings were recorded from all twenty gages after each incremental change in the circumference.

Determination of Elastic Compression Index of Grain

The following procedure was adopted in determining the elastic compression index of grain en masse. Figure 5 shows a specimen and the mold used in its preparation. Figure 6 shows the assembled test cylinder positioned under the loading yoke of the platform scale.

1. Place the rubber membrane over the pedestal in the center of the test cylinder and bind with a rubber band in tension.

2. Clamp the mold around the rubber membrane and fold the membrane down over the top of the mold.

3. Weigh a container of grain, pour the required amount of grain into the mold, and re-weigh the container.

4. Probe the specimen 25 times with a small rod to prevent the grain from clinging to the sides of the mold.

5. Place the cap on the surface of the grain. Remove the rubber membrane from the top of the mold and place it around the bottom of the cap. Bind the membrane with a rubber band in tension.

6. Apply a vacuum to the inside of the specimen and

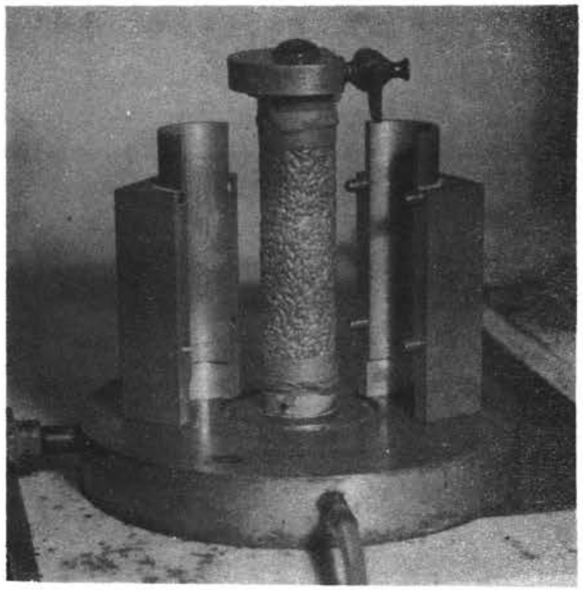


Figure 5. Specimen for Elastic Compression Index Test.

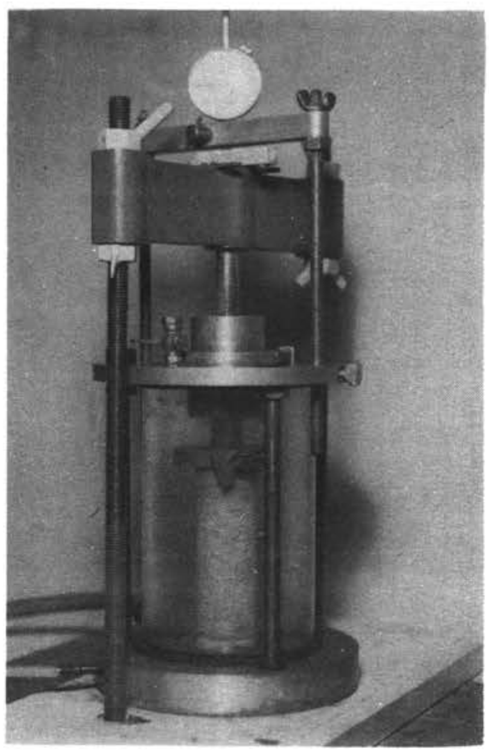


Figure 6. Triaxial Test Cylinder.

remove the mold as shown in Figure 5. Measure the height of the specimen.

7. Place the lucite testing chamber into position, screw the tie rods into the base, and set the head plate into position over the tie rods. See Figure 6.

8. Insert the well oiled loading piston through the head plate and carefully seat it on the ball in the center of the specimen cap. Tighten the nuts on the upper ends of the tie rods.

9. Center the test cylinder under the yoke of the platform scale and bring the loading head of the cross-bar into contact with the loading piston resting on the specimen.

10. Fill the test cylinder with water from the reserve-pressure cylinder allowing all the air to escape through a valve in the head plate.

11. Apply the desired air pressure to the water reserve-pressure cylinder and disconnect the vacuum source from the specimen.

12. Level the scale beam by adjusting the position of the beam rider to balance the upward thrust on the loading piston.

13. Arrange the dial indicator to measure the vertical displacement of the crossbar relative to the platform and record the initial dial reading.

14. Carefully place 10 gram load increments on the load hanger at one minute intervals and read the deformation of the specimen from the dial gage 55 seconds after the application of the load. Record the load increment and the

corresponding dial reading on the data sheet.

15. Continue loading the specimen until the deformation exceeds the 25 millimeter limit of the dial gage.

16. Remove the test cylinder from the platform scale and return the water to the water reserve-pressure cylinder.

17. Disassemble the test cylinder, remove the grain from the membrane, and prepare the equipment for the next test.

18. Repeat steps 1 through 17 three times for each retaining pressure used.

Determination of Modulus of Elasticity of Bin Materials

A device which utilized weights to apply a load through a level arm to a test specimen was used to determine the modulus of elasticity of the bin materials.

The specimens of bin materials were shaped to a one inch width at the test section. The two inch wide ends were reinforced on both sides with metal which was thicker than the medal being tested but was of the same type and grade. Three strain gages were mounted on one side of each of the specimens. Two specimens were used for the tests to determine the elastic properties of aluminum and one specimen was used for the test with galvanized steel. One replication was made with each specimen.

The modulus of elasticity (E) of the aluminum was found to be $13.8 \times 10^{+6}$ pounds per square inch. The data for the test on the galvanized steel was not consistent with known properties. Therefore, the value of $30 \times 10^{+6}$ pounds per square inch was used for the modulus of elasticity.

CHAPTER VI

ANALYSIS OF DATA

Strain Due to Active Pressures

Two replicated columns of strain gages were placed on each bin. Four replications of each test were made with each bin. Statistical analysis of strain data from preliminary experiments indicated there were no differences in strain due to differences between the strain gage column locations on the bin. There was no apparent reason to believe that the column location would affect the strain in the wall providing they were not in the area in which the bin wall was bolted together. The presence of slight variations in the thickness of the bin wall would cause a variation in the strain produced. However, thickness variations were believed to be randomized.

Figure 7 is a plot of the linear regression lines and the fitted curves of π_1 ($\mathcal{E}$) versus π_2 (h/H) for all six bins used in the test. Correlation coefficients are listed in the legend.

Lines 1, 2, 5, and 6 were obtained by linear regression only. However, the data at higher values of π_2(h/H) (near the bottom of the bins) for lines 3 and 4 did not conform to a straight line. This was thought to be the result of the effects of the ratio of the diameter of the bin to the depth of the grain. Previous investigations have borne out the fact that the lateral pressure ceases to increase appreciably after the depth of grain exceeds twice the diameter of the bin.

Line 3 was plotted by using the linear regression line from $\frac{h}{H}$ =

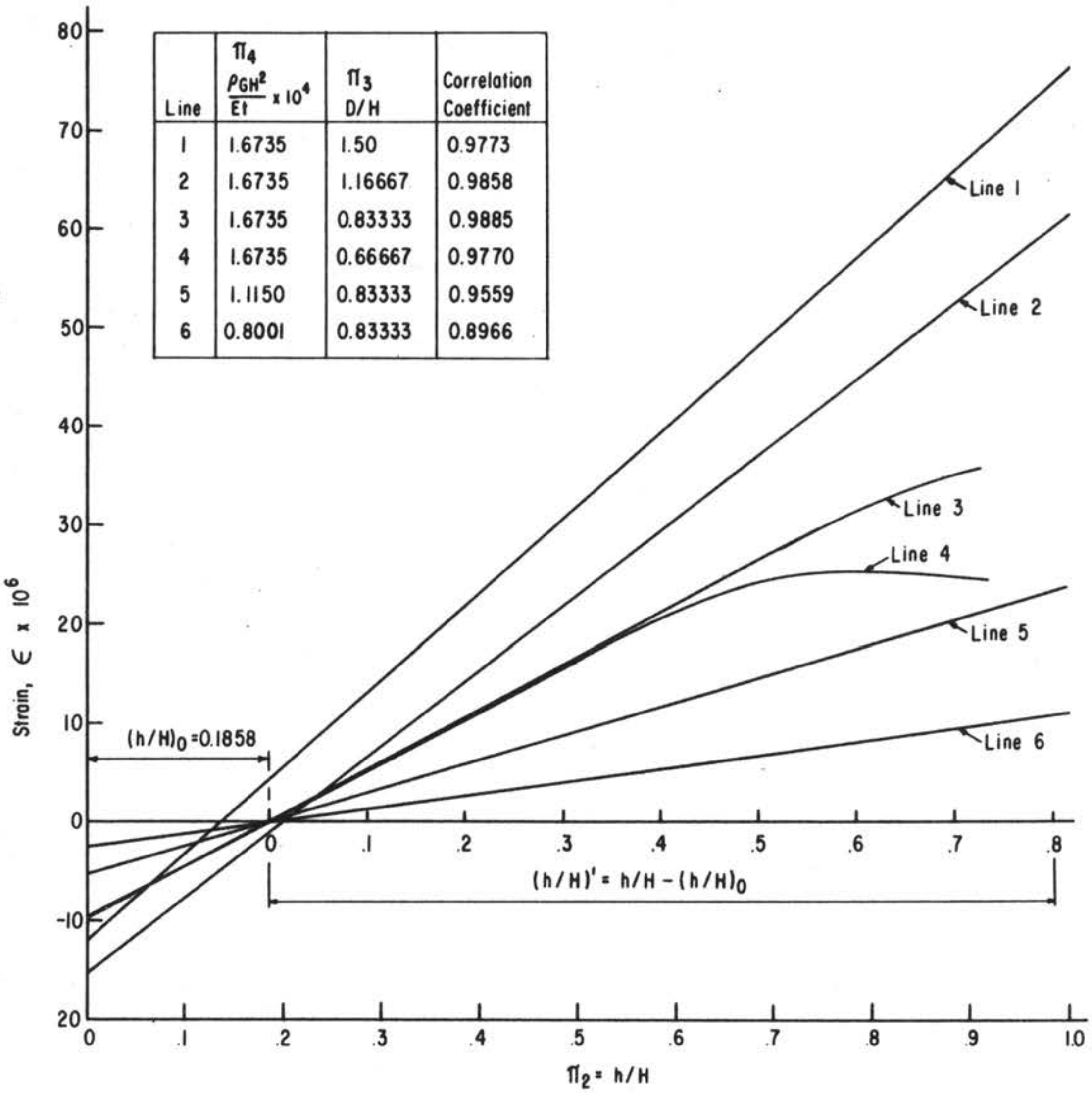


Figure 7. Effects of $\pi_2(h/H)$, $\pi_3(D/H)$, and $\pi_4(\rho GH^2/Et)$ on Strains Due to Active Pressures.

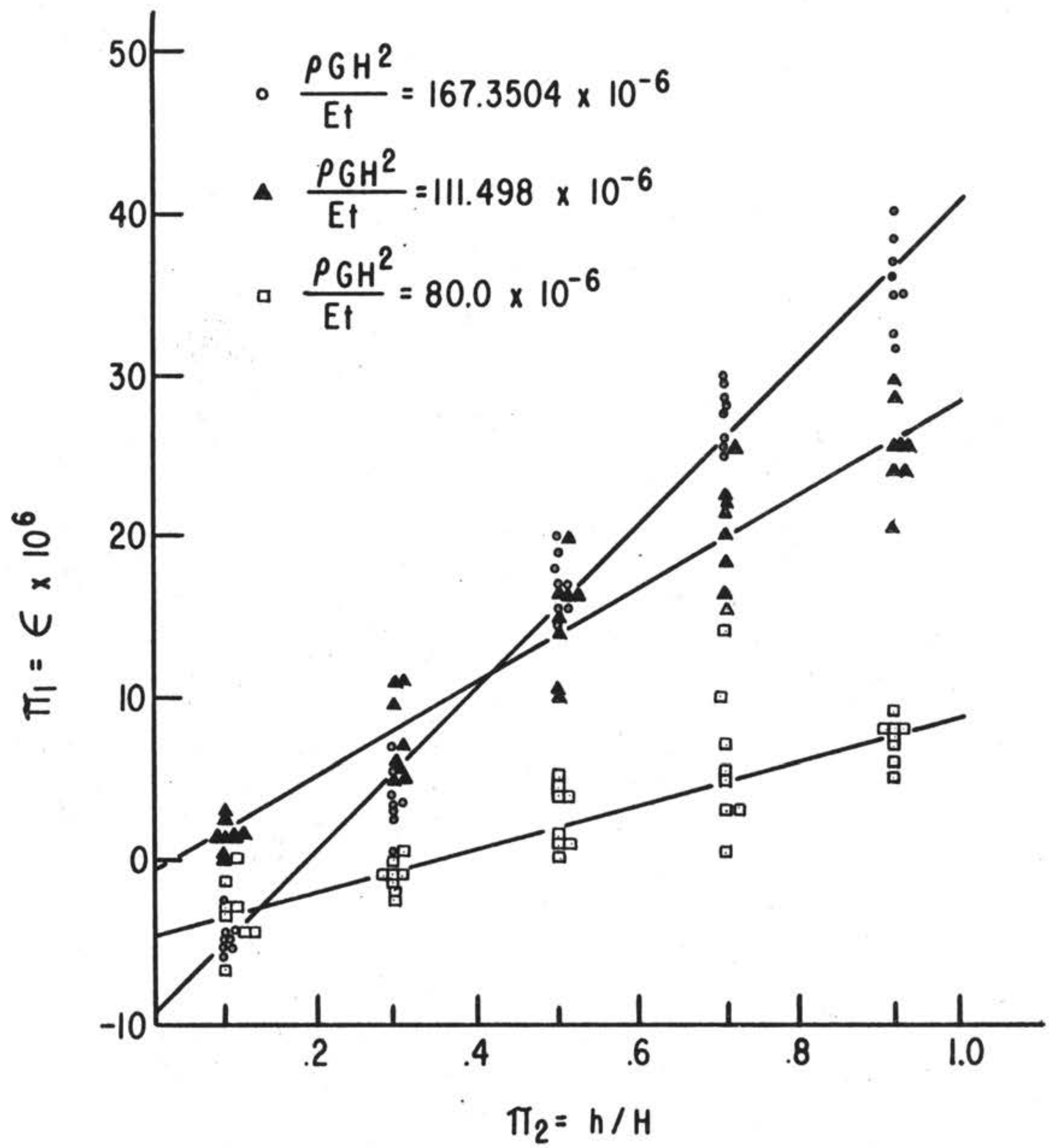


Figure 8. Plot of Experimental Data and Regression Lines for Strains Due to Active Pressures as Affected by $\pi_2(h/H)$ and $\pi_4(\rho GH^2/Et)$.

0.08333 to 0.70833. An average of the data for $\frac{h}{H} = 0.91667$ was plotted and a smooth curve was fitted to this point and the regression line at $\frac{h}{H} = 0.70833$.

Line 4 was plotted by using the linear regression line from $\frac{h}{H} =$ 0.08333 to 0.500. The averages of the data at $\frac{h}{H} = 0.70833$ and 0.91667 were plotted and a smooth curve was fitted to these points and the regression line at $\frac{h}{H} = 0.500$.

In Figure 7, lines 3, 5, and 6 cross the $\frac{h}{H}$ axis in the same location. The regression lines for the original data shown in Figure 8 were adjusted to intersect at this common point. The point was determined by taking the average of the values at which the original regression lines crossed the π_2 axis. The basic argument for this adjustment is that, when the bin is loaded with grain, the geometrical configuration of the cross-section of a bin wall should be similar for walls of different stiffnesses, i.e., the point of zero strain (no change in the diameter) should be at the same location on the wall providing the diameter - height ratio remains constant.

The point at which lines 3, 5, and 6 were adjusted to intersect the π_2 axis was used for the new origin of the abscissa. The new axis was designated as the π'_2 axis in Figure 7. This new axis was used in the remainder of the analysis. The relationship between the π_2 and the π'_2 axes was

$$\pi'_2 = \pi_2 - (\pi_2)_0 \qquad (6.1)$$

where $(\pi_2)_0$ was the point of intersection of each regression line on the π_2 axis.

Lines 3, 5, and 6 are straight when plotted on log-log paper and have a slope of unity. The intercepts of the lines on the log-log plot at $\pi'_2 = 1$ or Log $\pi'_2 = 0$ are equal in magnitude to the corresponding slopes of the lines plotted on arithmetic paper. Letting b represent the slope of line 3 on arithmetic paper, the following relationship can be established:

$$\pi_{1,\ \bar{3},\ \bar{4},\ \bar{5}} = b\ \pi_2' \tag{6.2}$$

where the bars denote constant values for the corresponding Pi terms.

The analysis to determine the relationship between π_1 ($\mathcal{E}$) and π_3 (D/H) was conducted with data obtained from regression lines 1, 2, 3, and 4. The strains for constant values of π_2' were calculated for these lines. Linear regression procedures were used for the logarithms of these numbers and the corresponding values of π_3 to determine the slope and the intercept of the regression line for each value of π_2'. An analysis to determine the homogeneity of the slopes indicated there were no significant differences among the slopes. The regression line for all the data passed through the family of lines at approximately $\pi_2' = 4.1$

The above procedure was used to determine the relationship between π_1 ($\mathcal{E}$) and π_4 $\left[\frac{\rho\, GH^2}{Et}\right]$. The data for the analysis were obtained from lines 3, 5, and 6. An analysis to determine the homogeneity of the slopes for constant values of π_2' indicated there were no significant differences among the slopes. The regression line for all the data passed through this family of curves at approximately $\pi_2' = 4.1$, also.

Since it was necessary to select a value of π_2' common to all of the components of the general prediction equation, $\pi_2' = 4.1$ was hypothesized to be the most appropriate value. The two above relationships become:

$$\pi_{1,\ \bar{2},\ \bar{4},\ \bar{5}} = 27.172838\ (\pi_3)^{0.9464} \tag{6.3}$$

$$\pi_{1,\ \bar{2},\ \bar{3},\ \bar{5}} = 9.0\ (\pi_4 \times 10^4)^{1.8084} \tag{6.4}$$

Equations 6.2, 6.3, and 6.4 are component equations for the general prediction equation. Line 3 was common to all the component equations. The strain at $\pi_2' = 4.1$ on this line was 21.746×10^{-6}.

Combining the component equations, the general prediction equation can be written:

$$\pi_1 = \frac{\left[53.04 \quad \pi'_2\right]\left[27.1728\, \pi_3^{\,0.9464}\right]\left[9.0(\pi_4 \times 10^4)^{1.8084}\right]}{(21.746)^2} \qquad (6.5)$$

Substituting equation 6.1 and simplifying, the equation becomes

$$\pi_1 = 27.4298 \left[\pi_2 - (\pi_2)_0\right] \left[\pi_3\right]^{0.9464} \left[\pi_4 \times 10^4\right]^{1.8084} \qquad (6.6)$$

There also is a relationship between $(\pi_2)_0$ and π_3 as indicated by the positions in which the regression lines cross the π_2 axis in Figure 7. This relationship was established by linear regression of the logarithms of $(\pi_2)_0$ and π_3. The equation for this system is:

$$(\pi_2)_0 = 0.175 \left[\pi_3\right]^{-0.2892} \qquad (6.7)$$

The correlation coefficient was only 0.6314. This could be expected because only four values were available for the analysis.

An analysis to determine the degree of fit of the prediction equation to the regression lines yielded a correlation coefficient of 0.9955, however, the slope of the regression line was 1.2764. The prediction equation was corrected by dividing by the slope of the regression line.

An analysis to determine the degree of fit between the corrected prediction equation and the observed data yielded a correlation coefficient of 0.9575 and the slope of the regression line was 0.9264. The prediction equation was again corrected by dividing by the slope of the regression line. The final prediction equation after substituting equation 6.7 into equation 6.6 and simplifying became:

$$\pi_1 = 26.2566 \left[\pi_2 - 0.175(\pi_3)^{-0.2892}\right] \left[\pi_3\right]^{0.9464} \left[\pi_4 \times 10^4\right]^{1.8084} \qquad (6.8)$$

It must be noted that this equation is valid only for bins containing wheat and which have the geometrical configuration and bin wall properties such that the Pi terms are within the ranges used for the present experiments and analysis.

Strain Due to Passive Pressures

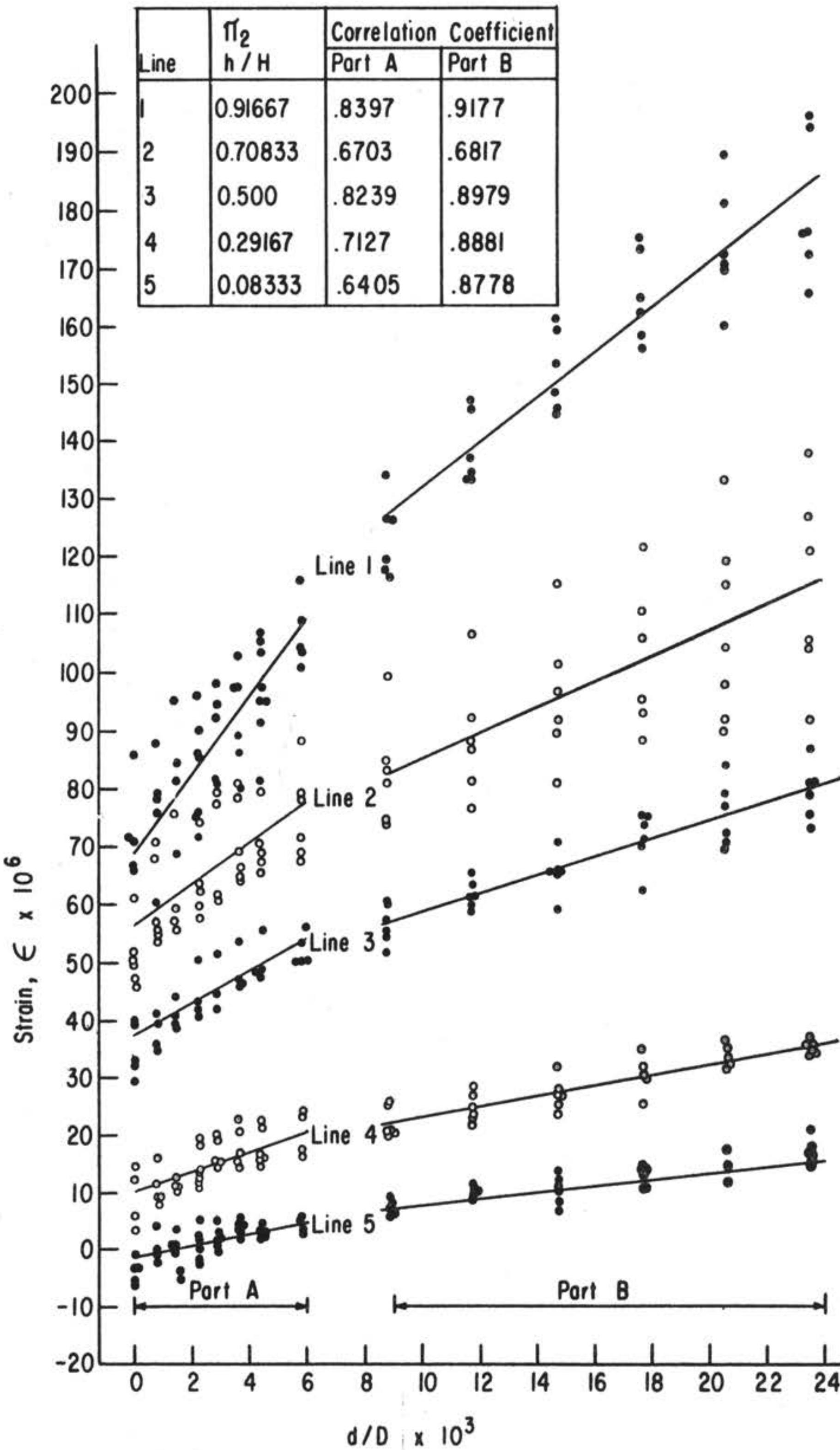
One model bin with two columns of strain gages was used for this test. The resulting direct strains were measured for fourteen increments of π_6 (d/D). Values of π_6 were coded by the factor 10^3 for convenience in analytical calculations.

The data for the elastic compression index of grain appeared to characterize two essentially different types of response. The data from the experiments on strain due to passive pressures followed a similar characteristic pattern at all depths in the bin. Two equations were therefore necessary to describe the system when passive pressures were produced. The data were divided into two parts, Part A (d/D = 0 to .007) and Part B (d/D = .007 to .0236), for the anlysis.

Linear regression procedures were used to analyze the date. Figure 9 shows the data and the regression lines for each value of π_2 (h/H). The correlation coefficients for the lines are shown in the legend.

The procedure for developing the equations was similar for Part A and Part B. The form of the equations of the regression lines was $\pi_1 = a + b\pi_6$, where a was the intercept on the $\pi_1(\mathcal{E})$ axis and b was the slope of the line. Since there were several lines, the intercept and the slope were functions of π_2.

A regression analysis of slopes of the lines versus π_2 yielded regression coefficients of 0.9424 for Part A and 0.9516 for Part B. The equation for this system was $b = b_0 + \beta\pi_2$, where b_0 was the intercept of the regression line and β was the slope. Also, the same procedure can be used to find the relationship of the intercepts, a, and π_2. The regression



Line	π_2 h / H	Correlation Coefficient Part A	Part B
1	0.91667	.8397	.9177
2	0.70833	.6703	.6817
3	0.500	.8239	.8979
4	0.29167	.7127	.8881
5	0.08333	.6405	.8778

Figure 9. Plot of Experimental Data and Regression Lines for Strains Due to Passive Pressures as Affected by π_2(h/H) and π_6(d/D).

analysis for this data yielded correlation coefficients of 0.9913 for Part A and 0.9928 for Part B. The equation for this system was $a = a_0 + \theta \pi_2$, where a_0 was the intercept of the regression line and θ was the slope.

The form of the general prediction equation becomes

$$\pi_1 = (a_0 + \theta \pi_2) + (b_0 + \beta \pi_2) \pi_6 \tag{6.9}$$

Substituting the values of a_0, θ, b_0, and β, which were found by linear regression, the equations which describe the system are:

For Part A

$$\pi_1 = \left[- 10.737 + 89.895 \pi_2\right] + \left[.0075 + 6.312 \pi_2\right]\left[\pi_6\right] \tag{6.10}$$

For Part B

$$\pi_1 = \left[- 13.135 + 111.387 \pi_2\right] + \left[-0.0133 + 3.719 \pi_2\right]\left[\pi_6\right] \tag{6.11}$$

where $\pi_6 = d/D \times 10^3$ and π_1 is strain in microinches per inch.

Elastic Compression Index of Grain En Masse

Three replications were made for each retaining pressure used. The retaining pressure varied from 5 to 30 pounds per square inch in increment of 5 pounds per square inch.

The ratio of the applied stress to the retaining pressure, $\frac{\sigma}{P}$, was computed for each increment of stress applied to the specimen. The corresponding strain, ε, was calculated by dividing the deformation of the specimen by the original height.

The information was plotted on log-log paper, as shown in Figures 10 and 11. The plots were divided into Parts A and B. The data for strains between 0.001 and 0.0015 were not included in the analysis and are not shown on the plots because they appeared to be in a transitional area between two systems of response, Part A and Part B, respectively.

The data for 5 pounds per square inch retaining pressure was not used

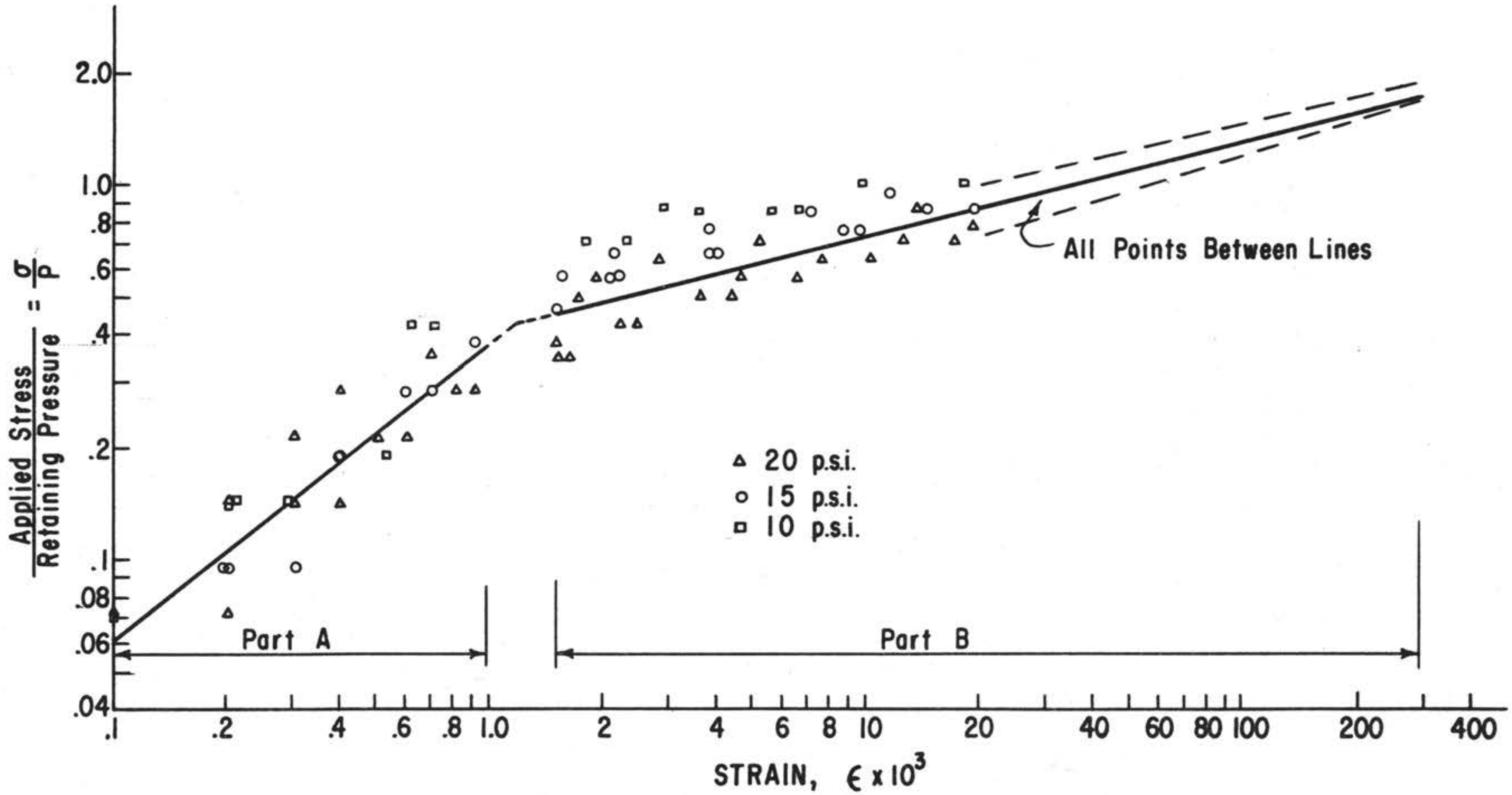


Figure 10. Plot of Regression Lines for 10-30 P.s.i. and Experimental Data for 10, 15 and 20 P.s.i. Retaining Pressures for Elastic Compression Index of Grain En Masse.

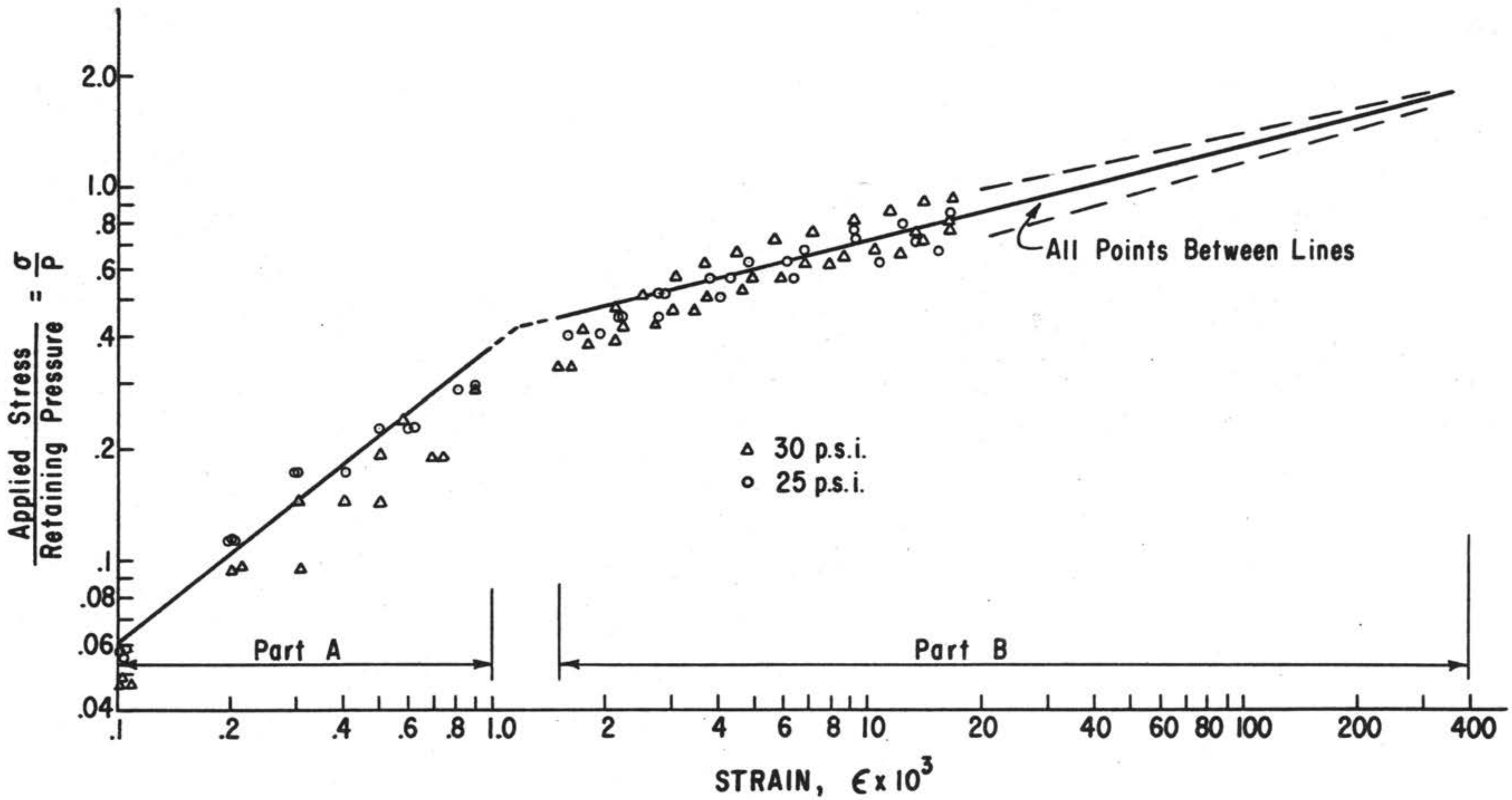


Figure 11. Plot of Regression Lines for 10-30 P.s.i. and Experimental Data for 25 and 30 P.s.i. Retaining Pressures for Elastic Compression Index of Grain En Masse.

in the analysis because the behavior pattern was not similar to that for retaining pressures of 10 pounds per square inch and greater. This was attributed to the application of relatively large loads to the specimen which caused large deformations and shearing in the area designated as Part A. The slope of the line for the data in Part B approached that of the data for higher retaining pressures.

Linear regression procedures were used to determine the slope and the intercepts of the regression lines for Part A and Part B. The logarithm of the dimensionless parameters, $\frac{\sigma}{P}$, and $\mathcal{E}$ were used in the analysis.

The correlation coefficients for this analysis were 0.913 for Part A and 0.9588 for Part B. The equations for the lines are:

For Part A;

$$\frac{\sigma}{P} = 87\ \mathcal{E}^{0.7948} \tag{6.12}$$

For Part B;

$$\frac{\sigma}{P} = 2.28\ \mathcal{E}^{0.2531} \tag{6.13}$$

The equation for Part A is applicable for strains up to 0.001 inch per inch and the equation for Part B is applicable for strains from 0.0015 to 0.25 inches per inch.

The exponents in the above equations were the values used for the elastic compression index, E_c, in subsequent data analysis.

CHAPTER VII

DISCUSSION OF RESULTS

The data obtained from the experiments dealing with active and passive pressures reveal that compressive strain occurs in the wall near the top of the bin. This can be explained by comparison with the reaction of a pipe which is loaded in compression around its circumference by a ring load, as presented by Den Hartog (1952). The diameter of the pipe will decrease in the vicinity of the load as shown in Figure 12. However, at some distance from the load, the diameter of the pipe will actually increase, and the pipe wall will be subjected to tensile hoop stresses, generated by compressive loading.

An analogy can be established between the pipe and the grain bin by considering the cylindrical grain bin as a large pipe and the grain as a semi-fluid with the equivalent load concentrated at one depth against the bin wall. The pipe was loaded in compression from the outside and the grain bin is loaded in compression from the inside. It follows that the diameter of the bin will increase in the vicinity of the equivalent grain load and at some distance from the load, near the top of the bin, the diameter will decrease. The point at which the diameter of the bin remains constant is a function of the diameter and the height of the bin.

Equation for Strain Due to Active Pressures

Equation 6.8 predicts the strain produced in the walls of cylindrical grain bins subjected to active pressures. The use of this equation is limited to systems which are geometrically similar to the model bins used

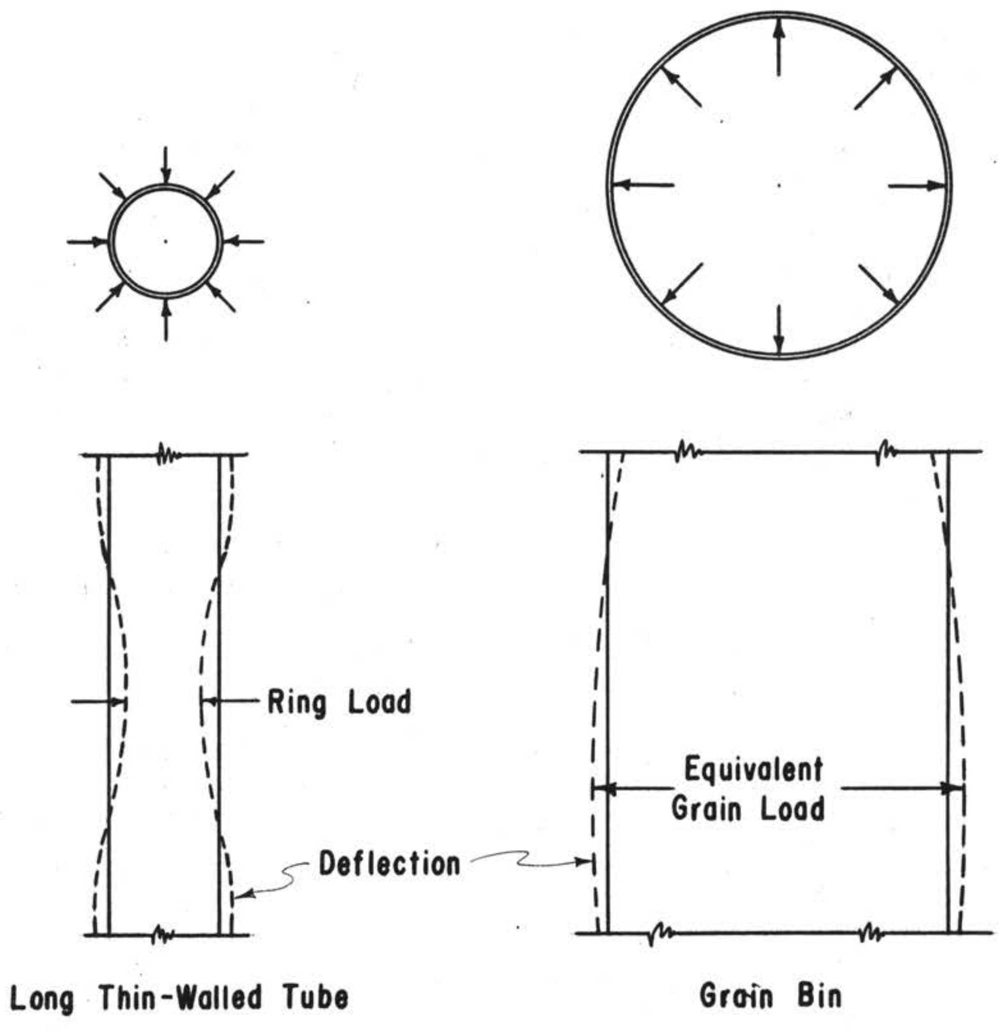


Figure 12. Deflections of a Pipe and a Grain Bin Wall When Subjected to Ring Loads.

in the experiments. Each of the dimensional parameters for prototype bins must be within the range used for the model bins.

A comparison of equations presently being used to predict strain in a bin wall and Equation 6.8 will show the differences between present design criteria and the results of this study. Airy's Equation and Rankine's Equation can be simplified as follows:

$$N = C(\rho Gh) \tag{7.1}$$

where C is a function of the coefficient of repose and the coefficient of friction for grain on a bin wall in Airy's equation and the angle of repose in Rankine's equation.

Applying thin-walled cylinder theory, the strain in the bin wall is:

$$\varepsilon = \frac{ND}{2Et} \tag{7.2}$$

substituting Equation 7.1 into Equation 7.2, the equation for strain becomes

$$\varepsilon = \frac{C}{2} \times \left[\frac{\rho GDh}{Et}\right] \tag{7.3}$$

Equation 6.8 can be written

$$\varepsilon = C_1 \left[\frac{h}{H} - f\left(\frac{D}{H}\right)\right] \left[\frac{\rho GhD}{Et}\right]^{1.8084} \left[\frac{D}{H}\right]^{-0.8616} \left[\frac{H}{h}\right]^{1.8084} \tag{7.4}$$

It is noted that the parameter in Equation 7.3 is of the first power. However, in Equation 7.4, the same parameter is raised to a power of almost two. Since Equation 7.3 is raised to the first power, it will predict only the direct influence of the stiffness of the wall on hoop strains. Equation 7.4 indicates that the stiffness of the wall affects the pressure produced on a grain bin wall. The pressures on the wall will determine the amount of strain produced in the bin wall. Neither Airy's equation nor Rankine's equations will predict the influence of the stiffness on the pressure produced on a grain bin wall.

It is also noted that Equation 7.4 contains parameters which determine the influence of the total height of the bin as related to the diameter of the bin and the depth below the grain surface.

Janssen's equation is widely used for designing grain storage structures. The equation can be simplified as follows:

$$N = C \ (\rho GD) \tag{7.5}$$

where C is a function of the type grain, the diameter of the bin, the depth below the surface of the grain, and the coefficient of friction of the grain on the bin wall. Substituting Equation 7.5 into Equation 7.2, the equation for strain becomes

$$\mathcal{E} = C \left[\frac{\rho GD^2}{Et}\right] \tag{7.6}$$

multiply by $\frac{h}{H}$ and $\frac{H}{H}$, Equation 7.6 becomes

$$\mathcal{E} = C \ x \left[\frac{\rho GhD}{Et}\right] x \ \frac{D}{H} \ x \ \frac{H}{h} \tag{7.7}$$

All the pertinent quantities considered in Equation 7.4 are also included in Equation 7.7. The stiffness parameter in Equation 7.7 is raised to the first power while the exponent for the same parameter in Equation 7.4 is 1.8084. It is noted that Janssen's equation will predict only the direct influence of the stiffness of the wall on hoop strains and will not predict the influence of the stiffness of the wall on the grain pressure produced on a bin wall.

Equation 6.8 is limited to predicting strains in small diameter bins. This is due to the range of π_4 $\left[\frac{\rho GH^2}{Et}\right]$ in the experiments.

Equation for Strain Due to Passive Pressures

Equations 6.10 and 6.11 predict strains which are produced when there is relative lateral movement of the bin wall or the grain mass. The equations are valid for those systems whose parameters are within the ranges used in the experiments. Equation 6.10 will predict strains in bin walls when the value of d/D is between 0 and .007. Equation 6.11 is to be used when d/D is between .007 and .0236.

The equations indicate that the strain will continue to increase indefinitely when the diameter of the bin is changed. However, the resistance of the grain to deformation would possibly reach a maximum after which the strain produced in the wall should be constant. The range of π_6 (d/D) in which the experiments were performed was not great enough to detect this.

Comparison of Equations

The plots of predicted strains versus h/H for Equation 6.8, Equation 6.10, Janssen's Equation, Airy's Equation, and Rankine's Equation are shown in Figure 13. The thin-walled cylinder theory was used in conjunction with Janssen's Equation, Airy's Equation, and Rankine's Equation to determine the strains in the bin walls.

A comparison was made between the strains due to active pressures predicted by Equation 6.8, Rankine's Equation, Airy's Equation, and Janssen's Equation. It was noted that grain bins designed by using Rankine's Equation would be over designed for tensile strains in the wall section where $0.1 < h/H < 0.63$ and under designed for compressive strains in the wall section where $0 < h/H < 0.1$ and under designed for tensile strains where $0.63 < h/H < 1.0$.

Grain bins designed by using Janssen's equation would be under designed for compressive strains in the wall section where $0 < h/H < 0.1$, under designed for tensile strains where $0.48 < h/H < 1.0$ and over designed for tensile strains in the wall section where $0.1 < h/H < 0.48$.

Grain bins designed by using Airy's Equation would be under designed for the entire bin wall except for the wall section where $0.1 < h/H < 0.26$.

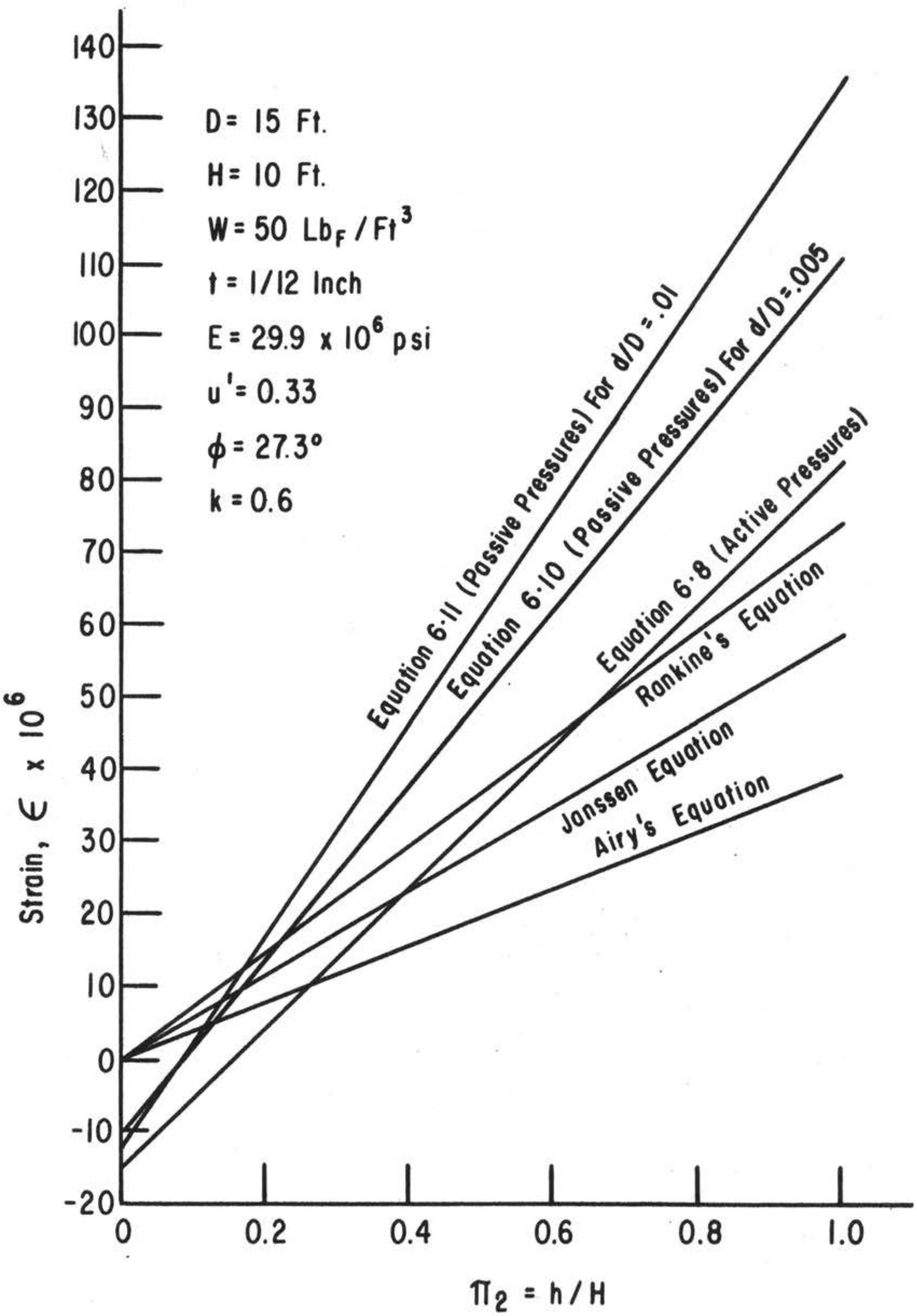


Figure 13. Comparison of Figures.

A bin wall subjected to passive pressures where $d/D = .005$ would be under designed if either Janssen's Equation, Rankine's Equation or Airy's Equation is used to determine the hoop strains in the wall.

Elastic Compression Index

The exponents in Equations 6.12 and 6.13 are the slopes of the regression lines for the experimental data of applied stress/retaining pressure (σ/P) versus strain (ε). They are indices of the elastic property of hard red winter wheat at 12.1 percent moisture content when subjected to a retaining pressure. The slopes of the regression lines are designated the Elastic Compression Indices of wheat en masse.

All the specimens began to creep (continuous deformation) at strains between .0035 and .0085 inch/inch and began to bulge at strains between .027 and .071 inch/inch. The point the specimens began to bulge was noted when there was an apparent visual shifting of grain particles.

The creeping and the bulging occurred in the system designated as Part B. See Figure 10 and 11. It was hypothesized that a grain mass would be completely elastic in Part A since creeping and bulging (particle rotation) were not present. However, when grain is subjected to a strain which causes rotation of the grain particles, the grain mass is no longer completely elastic. This is due to the friction between the grain particles which prevents the mass of grain from returning to its original shape.

Figure 14 shows a specimen after a test had been completed. The bulging on one side was typical for most specimens tested. The grain appeared to shear on two planes which allowed a wedge-shaped mass of grain to move laterally. The retaining pressure and the friction of grain on grain opposed the movement. There was no crushing of the grain particles

Figure 14. Specimen After Elastic Compression Index Test.

due to deformation of the specimen.

Vibration Test

Exploratory experiments were conducted to determine the effects of vibration on strains in grain bin walls. The tests were performed with the 2.5 foot diameter bin constructed of galvanized steel sheeting.

A small vibrating sander was attached to the frame of the top of the work table near the center of the bin. Vibrations from the sander did not cause movement of the model bin on the table top.

Vibration of the grain caused an increase in bulk density of the grain of approximately two percent. There was no increase in the strains in the bin wall. It was concluded that vibration of the grain mass did not produce strains in the walls similar to those produced by moving the wall against the grain mass. It was further decided that the only effect of vibrating the grain would be the effect of the increase in density on the active pressures.

Moore (1952) reported that vibration of granular materials increased the pressures on the side walls of a storage bin. However, the vibrating device used on the model bin could have caused the bin wall to vibrate or move while the grain was decreasing in volume. The increase in the strain in the wall would not be a direct result of the increase in density caused by vibration but would be a result of the wall moving and allowing the grain to become more dense. The "packed" grain then prevented the wall from returning to its original position.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

An experimental investigation was conducted to determine the effects of active and passive pressures on strains in cylindrical grain bin walls. Model bins used in the test were three feet in height and were constructed of metal sheeting. Lateral strains in the wall were measured directly by strain gages. Hard red winter wheat at 12.1 percent moisture content was used in the test. The principles of similitude were utilized in organizing and conducting the experiment.

Strains due to passive pressures were obtained by using a model bin with a device to change the circumference of the bin.

Tri-axial soils testing equipment was used to determine the elastic behavior of wheat en masse.

The following conclusions were drawn from the investigation:

1. Compressive strains exist in the wall near the top of the storage bin when the wall is subjected to active pressures and passive pressures. The maximum compressive strain produced at the top of the model bins used for this study was 16×10^{-6} inches per inch.
2. In the equation for active pressures developed from this study (Equation 6.8), the parameter which contains the stiffness of the wall is raised to the 1.8084 power. The same parameter is raised to the first power when either Janssen's Equation, Rankine's Equation or Airy's Equation

is used to predict the hoop strains in a bin wall. This implies that the stiffness of a bin wall affects the grain pressure produced on the wall.

3. The slope of the regression line of experimental data for applied stress divided by retaining pressure, σ/P, versus strain, ε, was designated the elastic compression index of grain en masse. The elastic compression index was 0.7948 for strains between zero and 0.001 inches per inch and 0.2531 for strains between 0.0015 and 0.27 inches per inch.

4. The equation for active pressures developed from this study (Equation 6.8) predicts strains at the bottom of a grain bin wall for D/H = 1.5 which are 11.5 percent greater than predicted by Rankine's Equation, 41 percent greater than predicted by Janssen's Equation, and 110 percent greater than predicted by Airy's Equation.

   Neither Rankine's Equation, Janssen's Equation nor Airy's Equation predict compressive strains near the top of the bin wall, which were found to exist in the present study.

5. The increase in lateral strains in a bin wall when passive pressures are produced is a function of the depth below the grain surface and the amount of lateral displacement of the bin wall. Strains due to passive pressures at the bottom of the model bin used in this test were as much as 2.7 times the strains due to active pressures.

6. The equations developed from data obtained from this investigation can be utilized only for geometrically similar

storage structures containing wheat. In addition, it is required that the numerical values of the dimensional parameters for the prototype be within the ranges used for the model bin study.

Suggestions for Further Investigations

1. Determine prediction equation for strains developed by other grains which are commonly stored in cylindrical structures.

2. Determine the affects of the parameters π_3 $\left[D/H\right]$ and π_4 $\left[\frac{\rho GH^2}{Et}\right]$ on the strains due to passive pressures and include these parameters in the prediction equation.

3. Determine the effects of moisture content, coefficient of friction of the grain on the bin wall, and the elastic properties of various grains on strains produced in bin walls and include these quantities in the prediction equations.

4. Determine the effects of temperature change and moisture content change on the expansion properties of grains when subjected to various retaining pressures.

5. Larger models should be used in future studies. The larger strains would increase the accuracy of measurements.

6. If larger models are used, a rosette type strain gage should be used to measure vertical and horizontal strains at a point. From these measurements, a more complete description of the state of strain could be obtained.

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A P P E N D I X A

SYMBOLS AND NOTATION

SYMBOLS AND NOTATION

d	Change in Diameter
D	Diameter of Bin
E	Modulus of Elasticity
E_c	Elastic Compression Index of Grain
F	Dimensional Symbol for Force
G	Gravitational Field Strength
h	Depth Below Grain Surface
H	Height of Bin Wall
k	Ratio of Lateral to Vertical Pressures in a Grain Mass
ℓ	Characteristic Particle Length
L	Dimensional Symbol for Length
m	Moisture Content
M	Dimensional Symbol for Mass
N	Unit Lateral Pressure
P	Retaining Pressure on Grain Mass
R	Hydraulic Radius
S	Characteristic Shape of Grain Particle
t	Thickness of Bin Wall
u	Coefficient of Repose, Tan ϕ
u'	Coefficient of Friction for Grain on Bin Wall
w	Unit Weight of Grain
α	Coefficient of Volumetric Expansion Due to Temperature Change
ε	Strain
η	Coefficient of Volumetric Expansion due to Moisture Content Change
θ	Inclination of Backfill Slope
ρ	Bulk Mass Density of Grain
σ	Stress Applied to Grain Mass
ϕ	Angle of Repose
$\triangle$	Temperature Change
ψ	Dimensional Symbol for Temperature

A P P E N D I X   B

EXPERIMENTAL DATA

STRAINS DUE TO ACTIVE PRESSURES

$\frac{\rho G H^2}{Et}$	$\frac{D}{H}$	Repli-cation	$\pi_2 = h/H$									
			0.08333		0.29167		0.5000		0.70833		0.91667	
			Col. I	Col. II	Col. I	Col. II	Col. I	Col. II	Col. I	Col. II	Col. I	Col. II
167.3504×10^{-6}	.66667	1	-4.0	-2.5	1.5	5.5	16.5	17.5	25.5	26.0	24.5	24.5
		2	-7.5	-6.0	2.5	5.5	17.0	20.0	26.5	25.5	23.0	28.0
		3	-6.5	-3.0	4.5	5.5	14.5	19.5	22.0	23.0	20.5	25.0
		4	-4.0	-4.0	5.0	4.0	14.0	18.0	23.0	26.5	23.0	27.5
	0.83333	1	-5.5	-6.0	3.0	0.5	17.0	14.5	25.0	26.5	36.0	31.5
		2	-5.0	-2.5	3.5	5.5	19.0	15.5	25.5	27.5	37.0	35.0
		3	-5.5	-4.5	7.0	4.0	20.0	17.0	29.5	30.0	35.0	32.5
		4	-5.0	-4.5	2.5	3.5	18.0	15.5	28.0	28.5	38.5	40.0
111.498×10^{-6}		1	1.5	1.5	11.0	7.0	14.0	20.0	25.5	18.5	25.5	24.0
		2	2.5	0.5	11.0	5.0	10.5	16.5	22.0	15.5	24.0	20.5
		3	3.0	0.0	9.5	5.0	10.0	15.0	22.5	20.0	28.5	25.5
		4	1.5	1.5	6.0	5.5	16.5	16.5	21.5	16.5	29.5	25.5
80.0×10^{-6}		1	-4.5	-3.5	-1.0	0.0	1.5	4.0	3.0	9.0	7.5	8.0
		2	0.0	-3.0	-1.0	-2.0	4.0	1.0	7.0	5.5	8.0	8.0
		3	-4.5	-7.0	0.5	-1.0	1.0	0.0	0.5	5.0	6.0	5.0
		4	-1.5	-5.5	-2.5	-1.5	5.0	4.5	3.0	10.0	7.0	9.0

Strains Due to Active Pressures (Continued)

$\frac{\rho GH^2}{Et}$	$\frac{D}{H}$	Replication	$\pi_2 = h/H$ 0.0833 Col. I	0.0833 Col. II	0.29167 Col. I	0.29167 Col. II	0.5000 Col. I	0.5000 Col. II	0.70833 Col. I	0.70833 Col. II	0.91667 Col. I	0.91667 Col. II
167.3504×10^{-6}	1.16667	1	-6.0	-6.5	3.0	8.5	27.5	19.0	35.0	34.5	60.0	54.0
		2	-10.0	-7.5	1.5	5.5	29.0	17.5	32.5	39.5	60.5	63.5
		3	-5.5	-5.5	5.0	5.5	25.0	20.0	34.5	34.0	56.0	53.0
		4	-5.5	-5.0	5.0	6.5	27.0	19.0	35.0	38.5	58.0	60.0
	1.50	1	0.0	-1.0	6.0	6.0	33.0	39.0	59.5	61.0	80.0	66.0
		2	-3.5	-3.5	6.0	3.5	31.5	29.0	50.0	45.5	71.0	61.0
		3	-6.5	0.0	14.5	10.0	40.0	32.5	51.5	47.0	70.5	59.5
		4	-1.5	-1.0	11.0	6.5	36.0	33.5	55.5	48.0	76.0	59.0

STRAINS DUE TO PASSIVE PRESSURES

$\frac{\rho G H^2}{Et} = 167.3504 \times 10^{-6}$ $\qquad\qquad$ $\frac{D}{H} = 1.500$

$\frac{d}{D} \times 10^3$	Replication	$\pi_2 = h/H$: 0.08333 Col. I	0.08333 Col. II	0.29167 Col. I	0.29167 Col. II	0.5000 Col. I	0.5000 Col. II	0.70833 Col. I	0.70833 Col. II	0.91667 Col. I	0.91667 Col. II
0.0	1	6.0	-1.0	12.0	6.0	39.0	39.0	65.5	61.0	86.0	66.0
	2	-3.5	-3.5	6.0	3.5	31.5	29.0	50.0	45.5	71.0	61.0
	3	-6.5	-3.5	14.5	10.0	40.0	32.5	51.5	47.0	70.5	49.5
0.7368	1	4.0	0.0	11.5	9.5	41.0	44.5	67.5	70.5	88.0	75.5
	2	0.0	-1.0	9.5	8.0	35.5	34.5	55.0	54.5	78.5	66.5
	3	-1.0	-2.5	16.0	16.0	44.0	39.0	56.5	53.5	79.0	60.0
1.4736	1	3.5	0.5	12.5	11.5	44.5	46.5	73.5	74.0	95.0	79.0
	2	0.5	-1.0	11.0	10.0	38.5	39.0	55.5	59.0	81.5	67.0
	3	-5.5	-4.0	13.0	15.0	44.0	40.5	57.0	55.0	84.5	68.5
2.2104	1	5.0	1.5	15.0	13.5	46.0	46.5	75.0	75.5	96.0	85.0
	2	2.5	0.0	11.5	12.5	41.5	40.5	59.5	62.0	85.5	71.0
	3	-2.0	-2.5	19.0	18.0	50.0	43.0	63.5	57.5	90.0	74.0
2.9473	1	5.0	1.5	15.0	14.5	48.5	48.5	77.0	79.0	98.0	92.0
	2	1.5	0.5	11.0	14.0	45.0	41.5	60.5	63.5	89.0	81.5
	3	3.0	-0.5	20.0	19.0	51.0	44.5	64.5	61.0	94.5	80.0
3.6841	1	4.0	1.5	15.0	14.5	49.0	50.5	78.5	81.0	103.0	97.5
	2	5.0	3.5	16.5	15.0	47.0	46.0	64.5	69.0	93.5	89.5
	3	4.0	1.5	22.5	20.0	53.5	46.0	66.0	63.5	97.5	85.5

Strains Due to Passive Pressures (Continued)

$\frac{\rho G H^2}{Et} = 167.3504 \times 10^{-6}$ $\frac{D}{H} = 1.500$

$\frac{d}{D} \times 10^3$	Replication	$\pi_2 = h/H$: 0.08333 Col. I	0.08333 Col. II	0.29167 Col. I	0.29167 Col. II	0.5000 Col. I	0.5000 Col. II	0.70833 Col. I	0.70833 Col. II	0.91667 Col. I	0.91667 Col. II
4.4209	1	2.5	2.0	15.5	14.5	49.5	51.5	79.0	81.0	105.5	107.0
	2	4.5	3.5	16.5	16.0	47.0	48.5	65.0	70.5	95.0	95.0
	3	2.0	3.0	22.5	21.5	55.0	48.0	68.5	67.0	103.5	91.5
5.8946	1	3.5	4.5	16.0	16.0	50.0	53.0	79.0	88.0	104.5	116.0
	2	5.5	4.5	17.5	18.0	50.0	50.0	67.5	78.5	104.0	107.5
	3	2.5	5.0	23.0	23.5	56.5	50.5	69.0	71.5	109.0	101.0
8.8419	1	5.5	6.5	20.5	19.5	54.0	59.5	84.5	99.0	118.0	134.5
	2	9.0	5.5	20.5	20.0	55.5	51.5	74.5	83.0	119.5	126.5
	3	6.0	8.0	25.5	25.0	60.0	57.0	74.0	80.5	126.5	116.5
11.7892	1	8.5	11.0	21.5	22.5	58.5	65.0	86.5	106.5	133.5	147.0
	2	9.5	9.0	24.5	23.0	61.0	60.0	81.0	92.0	137.0	145.5
	3	10.5	10.0	28.0	26.5	63.0	61.0	76.5	88.0	134.5	133.5
14.7365	1	6.5	13.5	23.5	25.5	59.0	70.5	91.5	115.0	148.5	160.5
	2	10.5	10.0	27.0	27.5	65.0	65.5	89.0	101.5	153.5	159.5
	3	12.0	8.0	31.5	26.5	65.5	65.5	80.5	96.5	144.5	145.5

Strains Due to Passive Pressures (Continued)

$\frac{\rho G H^2}{Et} = 167.3504 \times 10^{-6}$ $\qquad\qquad$ $\frac{D}{H} = 1.500$

$\frac{d}{D} \times 10^3$	Repli-cation	$\pi_2 = h/H$: 0.08333		0.29167		0.5000		0.70833		0.91667	
		Col. I	Col. II	Col. I	Col. II	Col. I	Col. II	Col. I	Col. II	Col. I	Col. II
17.6838	1	10.5	14.5	25.0	25.0	62.0	75.0	95.0	121.5	162.5	175.0
	2	14.0	10.5	30.0	29.5	70.0	73.5	92.5	110.5	164.5	173.0
	3	13.0	13.5	34.5	31.5	75.5	71.0	88.0	106.0	158.0	156.0
20.6311	1	14.5	17.0	32.0	31.5	70.5	83.5	104.0	133.0	169.5	189.5
	2	14.5	11.5	32.5	32.0	69.0	77.0	97.5	119.0	172.0	181.0
	3	15.0	17.5	36.0	34.5	72.0	79.0	89.5	115.0	160.0	170.5
23.5785	1	15.0	20.5	33.5	35.0	73.0	86.5	105.5	137.5	176.0	196.5
	2	16.5	14.5	34.5	34.0	78.5	81.0	104.0	126.5	172.0	194.5
	3	16.0	17.5	36.5	35.0	75.5	81.0	91.0	121.0	165.0	175.5

ELASTIC COMPRESSION INDEX

PART A

σ - Applied Stress

P - Retaining Pressure

ε - Strain in inches/inch

P = 10 Psi

$\overline{P}$	$\varepsilon \times 10^4$
0.1432	3.1496
0.2864	5.3464
0.4296	6.3963
0.1432	2.0292
0.2864	4.0587
0.4296	7.1028
0.1432	2.0612
0.2864	4.1225
0.4296	7.2144

P = 15 Psi

$\overline{P}$	$\varepsilon \times 10^4$
0.09548	2.0138
0.1910	4.0276
0.2864	7.0483
0.0955	3.1246
0.1910	5.2076
0.2864	7.2907
0.3819	9.3738
0.0955	2.0086
0.1910	4.0173
0.2864	6.0260

P = 20 Psi

$\overline{P}$	$\varepsilon \times 10^4$
0.0716	1.0094
0.1432	2.0189
0.2148	3.0284
0.2864	4.0379
0.3580	7.0664
0.0716	2.0451
0.1432	4.0903
0.2148	6.1355
0.2864	8.1807
0.0716	1.0387
0.1432	3.1163
0.2148	5.1939
0.2864	9.3490

P = 25 Psi

$\overline{P}$	$\varepsilon \times 10^4$
0.057285	1.0199
0.114570	2.0399
0.171856	4.0798
0.229141	6.1197
0.057285	1.0094
0.114570	2.0189
0.171856	3.0284
0.229141	6.0569
0.286427	8.0759
0.057285	1.0043
0.114570	2.0086
0.171856	3.0130
0.229141	5.0216
0.286427	9.0390

P = 30 Psi

$\overline{P}$	$\varepsilon \times 10^4$
0.047738	1.0069
0.095475	2.0138
0.143213	4.0276
0.190951	7.0483
0.047738	1.0094
0.095475	2.0189
0.143213	3.0284
0.190951	5.0474
0.238689	7.0664
0.286427	9.0854
0.047738	1.0199
0.095475	3.0598
0.143213	5.0997
0.190951	7.1396

ELASTIC COMPRESSION INDEX

PART B

σ = Applied Stress

P = Confining Pressure

ε = Strain in inches/inch

P = 10 Psi

$\bar{P}$	$\varepsilon \times 10^4$
0.7161	23.1942
0.8593	56.7900
1.0024	183.8241
1.1457	492.4855
1.2889	633.1679
1.4321	1267.2887
1.5753	1664.1391
1.7185	2374.9002
0.7161	23.3379
0.8593	65.9550
1.0024	175.5418
1.1457	407.9064
1.2889	545.9047
1.4321	748.8432
1.5753	1218.6459
1.7185	1975.6067
1.8618	2490.0560
0.7161	18.5513
0.8593	36.0720
1.0024	99.9711
1.1457	152.5332
1.2889	598.7962
1.4321	723.5024
1.5753	1143.9996
1.7185	1486.1689
1.8618	1963.3507

P = 15 Psi

0.4774	15.1036
0.5729	22.1519
0.6683	41.2832
0.7638	88.6078
0.8593	147.0084
0.9548	256.7613
1.0502	427.9356
1.1457	634.3516
1.2412	700.8075
1.3367	827.6778
1.4321	1178.0816
1.5276	1510.3610

P = 15 Psi

$\bar{P}$	$\varepsilon \times 10^4$
1.6231	2044.0219
1.7186	2547.4756
0.5729	15.6230
0.6683	21.8722
0.7638	38.5368
0.8593	70.8244
0.9548	116.6520
1.0502	357.2470
1.1457	541.5989
1.2412	645.7526
1.3367	797.8169
1.4321	1177.9777
1.5276	1333.1666
1.6231	1505.0202
0.5729	21.0911
0.6683	39.1692
0.7638	97.4208
0.8593	190.8243
0.9548	324.4014
1.0502	465.0088
1.1457	622.6900
1.2412	708.0588
1.3367	974.2085
1.4321	1512.5341
1.5276	1720.4322
1.6231	2360.1960

P = 20 Psi

0.5012	17.1613
0.5729	19.1802
0.6445	28.2656
0.7161	52.4934
0.7877	139.3095
0.8593	280.6379
0.9309	378.5584
1.0025	466.3840
1.0741	549.1621
1.1457	611.7504
1.2173	721.7847
1.2889	1094.2862

P = 20 Psi

$\bar{P}$	$\varepsilon \times 10^4$
1.3605	1225.5198
1.4321	1352.7155
1.5037	1874.6214
1.5753	2064.4054
0.3580	16.3615
0.4296	22.4971
0.5012	30.6779
0.5729	47.0395
0.6445	75.6723
0.7161	127.8249
0.7877	194.2938
0.8593	262.8080
0.9309	324.1640
1.0025	456.0793
1.0741	627.8760
1.1457	725.0230
1.2173	838.5315
1.2889	958.1756
1.3605	1307.9046
1.4321	1485.8369
1.5037	1723.0800
1.5753	2068.7186
1.6470	2160.7526
1.7186	2365.2725

P = 20 Psi

0.3580	15.5818
0.4296	24.9309
0.5012	42.5903
0.5729	65.4436
0.6445	103.8788
0.7161	174.5164
0.7877	244.1152
0.8593	300.2098
0.9309	363.5759
1.0025	519.3941
1.0741	717.8027
1.1457	801.9446
1.2173	945.2974
1.2889	1026.3228

P = 20 Psi

$\bar{P}$	$\mathcal{E} \times 10^4$
1.3605	1288.0975
1.4321	1584.1522
1.5037	1669.3328
1.5753	1999.6675
1.6470	2360.1271

P = 25 Psi

$\bar{P}$	$\mathcal{E} \times 10^4$
0.4010	19.3790
0.4583	28.5586
0.5156	40.7980
0.5729	64.2568
0.6301	100.9750
0.6874	155.0324
0.7447	214.1895
0.8020	283.5461
0.8593	373.3017
0.9166	445.7182
0.9339	510.9950
1.0311	583.4115
1.0884	723.1447
1.1457	872.0574
1.2030	995.4714
1.2603	1145.4041
1.3175	1382.0325
1.3748	1562.5637
1.4321	1932.8056
1.4894	2289.7882
1.5467	2475.4191
0.4583	20.1897
0.5156	29.2751
0.5729	42.3985
0.6301	61.5788
0.6874	92.8730
0.7447	139.3095
0.8020	196.8503
0.8593	272.5620
0.9166	312.9416
0.9339	336.1599
1.0311	364.4256
1.0884	410.8621
1.1457	479.5073
1.2030	719.7657
1.2603	973.1475
1.3176	1087.2198
1.3748	1388.0476
1.4321	1551.5848
1.4894	1717.1411
1.5467	2023.0163
1.6040	2299.6163
1.6613	2530.7894

P = 25 Psi

$\bar{P}$	$\mathcal{E} \times 10^4$
0.4010	16.0694
0.4583	21.0911
0.5156	28.1214
0.5729	38.1648
0.6301	48.2082
0.6874	69.2993
0.7447	93.4035
0.8020	125.5423
0.8593	168.7289
0.9166	217.9415
0.9339	371.6053
1.0311	555.3993
1.0884	588.5425
1.1457	615.6596
1.2030	652.8201
1.2603	740.1976
1.3176	1200.1847
1.3748	1280.5318
1.4321	1366.9050
1.4894	1788.7273
1.5467	2124.1764
1.6040	2427.4867

P = 30 Psi

$\bar{P}$	$\mathcal{E} \times 10^4$
0.3342	16.1105
0.3819	21.145
0.4296	27.1864
0.4774	35.2417
0.5251	46.3177
0.5729	59.4075
0.6206	79.5456
0.6683	105.7252
0.7161	137.9463
0.7638	165.1328
0.8115	250.7199
0.8593	322.2103
0.9070	389.6731
0.9548	449.0806
1.0025	539.7023
1.0502	608.1720
1.0980	665.5657
1.1457	719.9387
1.1934	803.5120
1.2412	1077.3909
1.2889	1228.4270
1.3367	1308.9796
1.3844	1378.4562
1.4321	1478.1400
1.4799	1874.8615
1.5276	2138.6712

P = 30 Psi

$\bar{P}$	$\mathcal{E} \times 10^4$
1.5753	2234.3274
1.6231	2418.5915
0.4296	17.1613
0.4774	21.1992
0.5251	25.2372
0.5729	31.2941
0.6206	37.3511
0.6683	45.4270
0.7161	57.5408
0.7638	72.6832
0.8115	92.8730
0.8593	115.0817
0.9070	140.3189
0.9548	171.6131
1.0025	207.9547
1.0502	277.6095
1.0980	533.0102
1.1457	606.7030
1.1934	665.2533
1.2412	759.1358
1.2889	822.7336
1.3367	911.5687
1.3844	1327.4782
1.4321	1432.4651
1.4799	1565.7177
1.5276	2057.3389
1.5753	2186.5536
0.3342	15.2992
0.3819	18.3591
0.4296	22.4389
0.4774	30.5985
0.5251	37.7381
0.5729	49.9775
0.6206	66.2967
0.6683	86.6957
0.7161	112.1945
0.7638	134.6334
0.8115	161.1521
0.8593	201.9501
0.9070	265.1870
0.9548	372.2818
1.0025	519.1546
1.0502	600.7506
1.0980	675.2070
1.1457	734.3641
1.1934	785.3616
1.2412	842.4788
1.2890	926.1148
1.3367	1074.0075
1.3844	1252.4988
1.4321	1401.4116

$\overline{P}$	$\mathcal{E} \times 10^4$
P = 30 Psi	
1.4799	1499.3268
1.5276	1663.5388
1.5753	1968.5039
1.6231	2231.6510

VITA

HARVEY E. HAMILTON

Candidate for the Degree of

Master of Science

Thesis: AN ANALYSIS OF LATERAL STRAIN IN CYLINDRICAL GRAIN BIN WALLS SUBJECTED TO ACTIVE AND PASSIVE PRESSURES

Major Field: Agricultural Engineering

Biographical:

Personal Data: Born at Shattuck, Oklahoma, January 18, 1936, the son of John and Bertha Mae Hamilton.

Education: Graduated from Shattuck High School, Shattuck, Oklahoma, in 1954. Attended Panhandle A. & M. College in 1954-1955; received the Bachelor of Science degree in Agricultural Engineering in May, 1960, from Oklahoma State University. Completed the requirements for the Master of Science Degree in May, 1964.

Experience: Served as a graduate research assistant for the Agricultural Engineering Department, Oklahoma State University, for one year.

Associate Member of American Society of Agricultural Engineers.